MARKING SCHEME

LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS

SUMMER 2016

INTRODUCTION

This marking scheme was used by WJEC for the 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS

MARK SCHEME - SUMMER 2016

	Additional Mathematics Summer 2016		Final
1	(a)(i)(7x +2)(3x - 2) (ii) -2/7 with 2/3 or -0.2857… or -0.286 with 0.666.. (b)(i) $(x+6)^2 \underline{+} \ldots$ …….. + 13 Least value (+)13 (ii) (x =) -6	B2 B2 B1 B1 B1 B1 8	B1 (7x - 2)(3x + 2) or 7x(3x -2) + 2(3x – 2) or (x - ⅔)(21x +6) or '(7x + 2) and/or (3x – 2)' or '(7x + 2) + (3x – 2)' or equivalent If a restart in (ii) to factorise, do not alter marking in (i), unless the candidate is clearly replacing their answer (i) Ignore sight of "=0" Must be from factorising. STRICT FT for their factors. B1 for each answer Do not accept from the use of the quadratic formula Sight of $(x+6)^2$ or $(x + {}^{12}/_{2})^2$ Ignore sight of '=0' Accept 49 - 36 if not evaluated, otherwise mark final value. Do not accept '= -13' or '=13' *$(x + 6)^2$ + 13, B1, B1 ISW.* Must follow completing the square FT their value but not 49 or - 36 FT from 'their $(x + 6)^2$' Do not accept (-6, 13)
2	(a) $36x^3 + 8x$ (+0) (b) $-8x^{-9}$ (c) ¾ $x^{-¼}$	B3 B1 B1 5	B1 for $36x^3$ (not $9\times4x^3$), B1 for +8x (not 4×2x), and B1 for +0 (or blank) provided at least 1 other mark awarded. Mark final answer Mark final answer Index needs to be simplified. Mark final answer
3	 { 35(3x) -14(x-6) + 10(2x+3)} (/70) {105x-14x+84+20x+30} (/70) (111x + 114)/70 or showing LHS ≡ RHS	M1 B1 B1 A1 4	Attempt to use common denominator, may be implied by sight of 35(3x) -14(x-6) + 10(2x+3) without sight of /70 May be seen in stages Or equivalent. May be seen in stages, as intention of method B1 for 1 slip (e.g. -84). Must be as a sum of 5 terms. Convincing **must** follow from fully correct working at each stage *If no denominator then possible M1 (see note above), B1 B1 A0, however if denominator replaced later all marks are allowable*
4	(a) (y+δy =) $(x+\delta x)^2 + 3(x+\delta x)$ Intention to subtract (y=) $x^2 + 3x$ to find δy (δy =) $2x\delta x + (\delta x)^2 + 3\delta x$ Dividing by δx **and** (lim) δx→0 $dy/dx = \lim_{\delta x\to0} \delta y/\delta x = 2x + 3$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working and notation All notation throughout the working must be correct in order to award the final A1 Do not accept $dy/dx = \lim_{x\to0} 2x + 3$ as a final answer *Use of dy/dx throughout max 4 marks only, final A0*

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5	(Area of circle(s) =) $(2\times)\ \pi\times4^2$ or $(2\times)16\pi$ (Curved surface =) $2\times\pi\times4\ \times 18$ or 144π (Area of card =) $(18 + 8 + 8)\times 2\times\pi\times4$ $(= 34\times 8\pi)$ (Area wasted =) $((18+8+8)\times 2\pi\times4) - (2\times\pi\times4\times18 + 2\times\pi\times4^2)$ (=) 96π (cm^2) or answer in the range 300 (cm^2) to 302(cm^2)	M1 M1 M2 m1 A1	 M1 for sight of $(\ldots+ 8 + 8) \times 2\times\pi\times4$ or $(18 + \ldots + \ldots) \times 2\times\pi\times4$ Intention to subtract areas of 2 circles and a rectangle from the card, depends on at least M3 previously awarded CAO. Accept an answer in the range provided supported by correct working *Alternative:* *(Width of card =) $2\times\pi\times4$ (= 25·1327 cm) M1* *(M0 if stated as 'area of the circle' unless clearly used as width of the card)* *(Area of the small rectangle =) $8\times2\times\pi\times4$ M1* *(Area of a circle(s) =) $(2\times)\ \pi\times4^2$ M1* *(Area wasted =) $2\times8\times2\times\pi\times4 - 2\times\pi\times4^2$ M2* *(or M1 for $8\times2\times\pi\times4 - \pi\times4^2$)* *(Area wasted =) 96π (cm^2) or 300 to 302 (cm^2) CAO A1*
	QWC2: Candidates will be expected to • present work clearly, with words explaining process or steps AND • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer QWC1: Candidates will be expected to • present work clearly, with words explaining process or steps OR • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer	QWC 2 8	QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar. QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar. QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.
6	(a) Multiplier $(5-\sqrt{2}) / (5-\sqrt{2})$ Denominator $25 + 5\sqrt{2} - 5\sqrt{2} - 2$ OR $25 - 2$ OR 23 $3(5-\sqrt{2})/23$ or $(15 - 3\sqrt{2})/23$ (b)(i) $x^{15/5}/x^{1/2}$ or $x^3/x^{1/2}$ $= x^{5/2}$ (ii) Correctly extracting a factor of $x^{1/9}$ (numerator) or $\frac{8x^{1/9}}{x^{2/9}} + 1\frac{(x^{2/9})}{(x^{2/9})}$ $\frac{8+x^{1/9}}{x^{1/9}}$ or $8x^{-1/9} + 1$ or $8/x^{1/9} + 1$	M1 A1 A1 B1 B1 M1 A1 7	Allow if the multiplier is stated as $(5-\sqrt{2})$ provided it is used as $(5-\sqrt{2})/(5-\sqrt{2})$ CAO. Mark final answer *Unsupported answer is awarded no marks.* Or equivalent first stage of working with indices CAO. Accept $x^{2.5}$ or $x^{2½}$ CAO. Mark final answer

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7	(a) $FG^2 = (-2\ -4)^2 + (14-6)^2\ \ (= 6^2 + 8^2)$ $FG = \sqrt{100}\ \ (= 10)$ (b) Gradient FG (14-6)/(-2-4) = 8/-6 (= - 4/3) (c) (-2 + 4)/2 or (14 + 6)/2 Mid point (1, 10) Perpendicular gradient ¾ (or 6/8) $\frac{y-10}{x-1} = \frac{3}{4}$ or 10 = ¾ × 1 + c y – 10 = ¾ (x - 1) or 4(y – 10) = 3(x – 1) or 4y = 3x + 37 or c = 9¼ or c = 37/4 4y – 3x – 37 = 0 or 3x – 4y + 37 = 0	M1 A1 M1 A1 M1 A1 B1 M1 m1 A1 10	Or equivalent. Allow 1 slip or error CAO Do not ignore incorrect cancelling, mark final answer Sight of (1, …) or (… , 10) implies M1 provided no incorrect working is seen FT -1/ 'their answer in (b)' OR for an alternative correct method of finding the equation of a straight line, for the idea of how an equation of a straight line can be found. FT 'their perpendicular gradient' or 'their answer in (b)' AND 'their mid point' or for 'points F or G' used **Do not allow** for use gradient from their answer in (b), and/or points F or G. Only FT for 'their perpendicular gradient' (with B1 previously awarded) AND 'their mid point' CAO. Must be in this form with '=0'
8	(dy/dx=) $3x^2 - 6x$ dy/dx = 0 or $3x^2 - 6x = 0$ or $3x^2 = 6x$ x = 0 and y = 11 x = 2 and y = 7 $d^2y/dx^2 = 6x - 6$ (0, (11)): $d^2y/dx^2 < 0$, point is a maximum (2, (7)): $d^2y/dx^2 > 0$, point is a minimum	B1 M1 A1 A1 M1 A1 A1 7	 FT their dy/dx form $ax^2 \pm bx$ *Answer only, no working shown M0 A0 A0* Or first derivative test, interpretation of first derivative test. Or alternative (e.g. full graphical method with explanation) FT for their x value FT for their other x value provided this does not have the same interpretation as the first x value *Answer only, no working shown M0 A0 A0* *If $d^2y/dx^2 = cx + d$ where $c \neq 0$ and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded))*
9	(a) $\frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$ (b) ($\frac{½}{\sqrt{3}/1}$ =) $\frac{1}{2\sqrt{3}}\ \frac{(\times\sqrt{3})}{(\times\sqrt{3})}$ or $\frac{½\sqrt{3}}{3}$ $= \sqrt{3}/6$ (c) $(\sqrt{3}/2)^2 + 1^{(2)}$ 7/4 or 1¾ or 1.75	B1 M1 A1 M1 A1 5	**<u>Working must be shown throughout</u>** Must be sight of 1/√2 / 1/√2 or √2/2 / √2/2 or cos45°/sin45° = 1/tan45° Must be sight of $\frac{½}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ leading to $\frac{½\sqrt{3}}{3}$ Do not accept ½ ÷ √3 This stage must be seen, do not accept starting with ¾ + 1, this is M0, A0

	Additional Mathematics Summer 2016		Final
10	(a) $(4)^3 + 6(4)^2 – (4) - 30$ (= 64 + 96 – 4 - 30) = 126 (b)(i) Substitute x = 2 Showing f(2) = 0 (ii) $(x-2)(x^2 + bx + c)$ or intention to divide by (x-2) with x^2 shown ((x - 2)) $(x^2 + 8x + 15)$ ((x - 2))(x + 3)(x + 5)	M1 A1 M1 A1 M1 A2 A1 8	Or division method giving $x^2 + 10x$ … Or division method giving $x^2 + 8x$ … Convincing, working shown Allow $2^3 + 6(2)^2 - (2) – 30 = 0$ A1 for +8x or +15. Or use of factor theorem A1 (x+3), A1 (x+5) CAO. Mark final answer, *but ignore attempts to 'solve'*
11	(a) Correct shaped graph with (0°,) 180° & 360° labelled on the x-axis AND 2 & 8 labelled on the y-axis (b) Maximum value 8 AND Minimum value 2	B3 B2 5	*Intention for approximately (90°, 5) and (270°, 5)* B2 awarded a for correct shape graph with conditions: • cos*x* reflected • with one complete period, labelled 0° to 360° • with difference in y values between maximum and minimum of 6, for their labels OR B1 for a correct shape graph with any two of the 3 bullet points above met *If no marks, award SC1 for a curve through at least 5 correct points across the full range with all other conditions met. Do not accept a parabola or straight lines* Accept Maximum (180(°), 8) and Minimum (360(°), 2) Allow unsupported correct responses FT provided at least B2 previously awarded in (a) B1 for either value correct, or for a difference between their max and min of 6, or if their answers are reversed (including Maximum (180(°), 2) and Minimum (360(°), 8))
12	(a) (dy/dx=) $21x^6 + 4$ $(d^2y/dx^2=)$ $126x^5$ (b) (4/4) x^4 + $(2/2)x^2$ + $(4/-1)\ x^{-1}$ + c (constant) (c) $8x^2/2 + 2x$ $[\ 8x^2/2 + 2x\ ]^3_2$ and with intention to substitute and subtract $=(8\times3^2/2+ 2\times3) – (8\times2^2/2+ 2\times2)$ (= 42 – 20) = 22	B1 B1 B3 B1 B2 M1 A1 A1 11	Accept sight of $21x^6 + 4$ FT to 2nd B1 from $dy/dx = kx^n$ (+ m) B1 for each term. Accept unsimplified. ISW Award if at least B1 given for integration B1 for $8x^2/2$ or 2x Intention to use 3, 2 (in either order) **and** subtract FT their integration, not the same terms as given or differentiated, this includes if there is only1 term seen. FT for correct use of limits provided working with 2 terms from 'their integration' CAO, not FT. *Answer only, no working shown, M0 A0 A0*

	Additional Mathematics Summer 2016		Final
13	(When $x = 3$) $y = 33$ (Gradient when $x = 3$, $dy/dx =$) $3\times2x$ 18 Equation $\frac{y - 33}{x - 3} = 18$ or $33 = 18\times3 + c$ $y - 33 = 18(x - 3)$ or $c = -21$ $y = 18x - 21$ or equivalent	B1 M1 A1 M1 m1 A1 6	 For differentiation, before substitution of $x = 3$ FT values for 'their 33' and 'their 18' provided at least one of these is correct. Implies previous M1 CAO. Mark final answer
14	Method to solve simultaneously, e.g. use of $y = 10 - x$ or $x = 10 - y$ into the first equation $x^2 - 5x + 4 = 0$ or $y^2 - 15y + 54 = 0$ $(x - 4)(x - 1)$ (=0) or $(y - 9)(y - 6)$ (=0) (4, 6) and (1, 9)	M1 A1 m1 A1 4	$10 - x = x^2 - 6x + 14$ or $y = (10 - y)^2 - 6(10 - y) + 14$ Must '=0' or implied in further working OR $x = (5 \pm \sqrt{9})/2$ or $y = (15 \pm \sqrt{9})/2$ FT from their quadratic CAO Need not be in this form, accept x=4, y=6 with x=1, y=9 x & y values must be given Do not accept unsupported responses Do not accept trial & improvement
15	(a) Intention to substitute x=2 and x = 4 into $y = -x^2 + 6x - 8$ Showing y = 0 for both values (b) Intention to integrate $-x^3/3 + 6x^2/2 - 8x$ Use of correct limits 4 & 2 in the correct order **and** intention to subtract 4/3 or 1.33(3…)	M1 A1 M1 A2 m1 A1 7	OR substituting either value and showing y = 0 OR attempt to factorise as a pair of brackets ((-)x ..2)(x ..4) Do not accept $(-2)^2 + 6\times2 - 8$ and $(-4)^2 + 6\times4 - 8$ Accept $-2^2 + 6\times2 - 8$ and $-4^2 + 6\times4 - 8$ OR factorised as $(-)(x - 2)(x - 4)$ or equivalent Intention to integrate (manipulated given, hence not using given or differentiated) A1 one term correct. The limits must be used in the correct order CAO. Only allow 1.3 from correct working and sight of 4/3 *Answer only gets no marks* *No marks for use of the trapezium rule*