MARKING SCHEME

LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS
9550/01

SUMMER 2017

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS

MARK SCHEME - SUMMER 2017

	Additional Mathematics Summer 2017		**Final Version**
1	(5x + 3)(4x - 1) -3/5 and 1/4	B2 B2 4	B1 (5x… 3)(4x … 1) or (5x…1)(4x…3) or 5x(4x -1) + 3(4x -1) or 4x(5x + 3) – 1(5x + 3), or (20x +12)(x - ¼) B0 for (5x + 3)(20x -5) Ignore sight of "=0" Must be from factorising, do not accept use of quadratic formula followed by 'factorising'. **MUST FT for their factors** FT for 'their factors' equivalent difficulty not leading to whole number solutions. B1 for each answer
2	(a) $70x^9$ - 5 (+0) (b) $-12x^{-13}$ or $-12/x^{13}$ (c) $\frac{3}{8}x^{-5/8}$ or equivalent (d) $-4x^{-5}$ or $-4/x^5$	B3 B1 B1 B1 6	B1 for $70x^9$ (not $10\times7x^9$), B1 for -5 and B1 for +0 (or blank) provided at least one other mark awarded. If B3 penalise further incorrect working -1 CAO, although ISW. Index needs to be simplified. CAO, although ISW. Index needs to be simplified. CAO, although ISW. Index needs to be simplified. Penalise including '+c' -1 only
3	$(x+11)^2$ (± …) or $(x+22/2)^2$ (± …) (Minimum value at x =) -11 (Minimum value is) (+) 2	M1 A1 A1 3	Ignore 'their (± …)' or '=0' Do not accept method dy/dx =2x+22 CAO. Must be from sight of completing the square CAO. Must be from sight of completing the square
4	(a) $2(-4)^3 - 5(-4)^2 + 8(-4) - 6$ (= -128-80-32-6) -246 (b)(i) Substitute x = 2 Showing f(2) = 0 (ii) $(x-2)(x^2 + bx + c)$ or intention to divide by (x-2) with x^2 shown ((x-2)) $(x^2 + 11x + 30)$ ((x-2)) (x +5)(x + 6)	M1 A1 M1 A1 M1 A2 A1 8	Or division method giving $2x^2 - 13x$ … Or division method giving $x^2 + 11x$ … Accept sight of substitution with '=0' shown Or equivalent ie. $X^2 + 11x - 60$ with no remainder If any values are inserted at least 1 needs to be correct, appropriate sight of +11x or +30 implies M1 (and A1 to follow) A1 for (+)11x or (+)30 Or use of factor theorem A1 (x+5), A1 (x+6) CAO, but ignore sight of "=0", ISW

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5	$3x - 1 = 4x^2 + 8x - 3$	M1	
	$4x^2 + 5x - 2 = 0$	A1	Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0 FT provided their quadratic does not factorise and equivalent level of difficulty
	$x = \{-5 \pm \sqrt{(5^2 - 4\times4\times-2)}\}/2\times4$	m1	Use of correct quadratic formula, allow 1 slip in substitution (not a slip with the formula) If completing the square used award m1for sight of $(2x + 1.25)^2 \pm \ldots$ or $4(x + 5/8)^2 \pm \ldots$
	$x = \{-5 \pm \sqrt{57}\}/8$	A1	If m0 then A0 (leading to $x = (-1.25 \pm\sqrt{3.5625})/2$)
	$x = 0.318\ldots$ or $x = 0.32$ **and** $x = -1.5687\ldots$or $x = -1.57$	A1	If m0 then A0
	$x = 0.32$ with $y = -0.04$ **and** $x = -1.57$ with $y = -5.71$	A1	FT provided M1, m1 previously awarded using their values of x in 3x - 1 or equivalent to find y-values to 2 d.p. <u>Use of 2d.p. x values in the quadratic leads to y = -0.03 and y = -5.70</u> *Alternative using x = (y + 1)/3* M1 $y = 4\left(\frac{y+1}{3^2}\right)^2 + 8\left(\frac{y+1}{3}\right) - 3$ or equivalent A1 $4y^2 + 23y + 1 = 0$ or equivalent (equate to zero) m1 $y = \{-23\pm\sqrt{(23^2 - 4\times4\times1)}\}/2\times4$ or equivalent Allow 1 slip in substitution A1 $y = (-23\pm\sqrt{513})/8$ or equivalent A1 $y = -0.04(38\ldots)$ and $y = -5.7(061\ldots)$ A1 $x = 0.32, y = -0.04$ with $x = -1.57, y = -5.71$ FT to final A1, provided M1, m1 previously awarded using their values of y in (y + 1)/3 or equivalent to find x-values to 2 d.p.
		6	

	Additional Mathematics Summer 2017		Final Version
6	(a) (FG^2 =) $(20-10)^2 + (8 - -4)^2$ $(=10^2 + 12^2)$	M1	Or equivalent. Allow 1 slip in sign of substitution Allow for sight of $10^2 + 12^2$
	FG = √244	A1	CAO
	= 2√61	B1	FT 'their FG' of equivalent difficulty expressed correctly, e.g. √104 = 2√26, or √44 = 2√11needs to be in the form a√b where a≠1 and b ≠ 1 or simpler Sight of 2√61 implies previous √244
	(b) Gradient FG (20 – 10) / (8 - - 4)	M1	Or equivalent
	= 10/12 (= 5/6)	A1	CAO. Mark final answer and then FT
	Gradient perpendicular -12/10 (= -6/5)	B1	FT -1/grad FG
	(8+ - 4)/2, (20 + 10)/2	M1	Accept (2, …) or (… , 15)
	Mid point FG (2, 15) or equivalent	A1	CAO
	Use of y=mx+c or $\frac{y-y_1}{x-x_1}= m$	M1	Must show substitution of 3 values Method to find the equation using mid-point and perpendicular gradient (not 10/12 or 5/6 or 'their gradient') FT their mid-point (not F or G) & their perpendicular gradient (not 10/12 or 5/6 or 'their gradient'), or FT substitution of their midpoint with their perpendicular gradient (not 10/12 or 5/6 or 'their gradient'), in y = mx + c (towards finding c) *If no working for finding gradient is seen, then 'their 'spurious' incorrect perpendicular gradient' must be negative*
	y = -12x/10 + 17⅖ or y – 15 = -12/10(x -2)	A1	FT for correct unsimplified form, not written in quotient form, i.e. $\frac{y-15}{x-2} = \frac{-12}{10}$
	6x + 5y – 87 =0 OR -6x - 5y + 87 = 0	A2	Accept terms in different orders provided '=0' CAO for A2 and A1 A1 for 12x + 10y – 174 = 0 or other multiple of the correct response (not with fractional coefficients), with integer values for a, b and c, and with terms in any order provided '=0', OR A1 for 5y = - 6x + 87 or equivalent correct simplified equation but not given in the required form
		12	

	Additional Mathematics Summer 2017		Final Version
7	(Arc length =) $2 \times \pi \times 5 \times 110/360$ $= 55\pi/18$ (cm) (Circumference) $55\pi/18 = 2 \times \pi \times$ cone radius or equivalent (Cone radius =) 55/36 or 1.527(77…cm) (Perpendicular height^2 =) 5^2 – cone radius^2 (Perpendicular height =) 4.7608…(cm) (Volume of the cone =) $\frac{1}{3} \times \pi \times$ cone radius^2 × perpendicular height (=) 11.6 (cm^3)	M1 A1 M1 A1 M1 A1 M1 A1	 (= 9.599…cm) May be implied in later working FT 'their derived arc length' Allow for $55\pi/18 = \pi \times$ diameter, if clearly diameter FT 'their derived cone radius', provided ≠ 5 FT 'their derived cone radius' and 'their derived perpendicular' provided neither value ≠ 5 May be shown in stages, e.g. method to find area first CAO. Award M1 only for 11.636… cm^3 or 12 cm^3 *Alternative (for1st 4 marks) (not requiring arc length)* *(Area of sector =) $\pi \times 5^2 \times 110/360$ with intention to find the surface area of the cone* *M1* *$= 275\pi/36$ (cm^2) (=23.998..cm^2)* *A1* *(Surface area of cone $= \pi rl$)* *$275\pi/36 = \pi \times$ cone radius × 5* *M1* *(cone radius =) 55/36 or 1.527(77…cm)* *A1*
	QWC2: • Candidates will be expected to present work clearly, with words explaining process or steps AND • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer QWC1: Candidates will be expected to • present work clearly, with words explaining process or steps OR • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer	QWC 2 10	QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar. QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar. QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.
8	(a) $1140x^{18}$ (b) a = 3 b = 2 c = -6	B2 B1 B1 B1 5	B1 for sight of $60x^{19}$. FT to 2nd B1 from $dy/dx = kx^n$ B0 for 60^{19} or 1140^{18} Ignore incorrect notation *Accept sight of correct answers from 'uncorrected' working* *Do not accept embedded answers, candidates need to identify values for a, b and c, not accept as left in working without clearly stating.*
9	$x = 5 \times \tan 60°$ or $\tan 60° = x/5$ (x =) $5\sqrt{3}$	M1 A1 2	OR for use of 30°, 60° triangle with 5× sides of 1, 2 & $\sqrt{3}$ Do not award A1 unless a line of working is seen, i.e. $5\sqrt{3}$without working is M0, A0

	Additional Mathematics Summer 2017		Final Version
10	(a) $10x^5/5 + 24x^3/3 - 2x + 3x^{-3}/-3$	B4	B1 for each term ISW from correct unsimplifed form. Simplified form is $2x^5 + 8x^3 - 2x - x^{-3}$
	+ c (constant)	B1	Awarded only if at least B1 is awarded for integration
	(b) $12x^4/4 + 6x^3/3$	B2	Mark final answer, then FT. B1 for sight of $12x^4/4$ or $6x^3/3$ Ignore inclusion of '+c' throughout' (except final A mark)
	$[12x^4/4 + 6x^3/3]^2_1$	M1	FT their integration, not original. Intention to use 2, 1 **and** subtract
	$=(3\times2^4 + 2\times2^3) - (3\times1^4 + 2\times1^3)$	A1	FT for correct use of limits Accept unsimplified fractions included
	= 59	A1	CAO, not FT. Do not accept '59 + c' *Answer only, no working shown M0 A0 A0*
		10	
11	(dy/dx=) $9x^2 + 18x$	B1	
	dy/dx = 0 or $9x^2 + 18x = 0$	M1	FT their dy/dx form $ax^2 + bx$ throughout
	x = 0 and y = 4	A1	
	x = -2 and y = 16	A1	*Answer only, no working shown M0 A0 A0* *Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0*
	$d^2y/dx^2 = 18x + 18$	M1	Or first derivative test, interpretation of first derivative test. Or alternative.
	At (0, 4) $d^2y/dx^2 > 0$, point is a minimum	A1	FT for their x value (ignore y-values)
	At (-2, 16): $d^2y/dx^2 < 0$, point is a maximum	A1	FT for their other x value provided this does not have the same interpretation as the first x value (ignore y-values) *SC1 for correct FT from $d^2y/dx^2 = ax + b$, a>0, including allowed FT from $d^2y/dx^2 = 18x$, with x=-2 as maximum and x=0 as a minimum (despite $d^2y/dx^2 = 0$)* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
		7	
12	$y+\delta y = (x+\delta x)^2 + 10(x+\delta x)$	B1	Or alternative notation. Allow if final bracket omitted
	Intention to subtract (y=) $x^2 + 10x$ to find δy	M1	
	$\delta y = 2x\delta x + (\delta x)^2 + 10\delta x$	A1	Accept δx^2 as meaning $(\delta x)^2$
	Dividing by δx **and** (lim) $\delta x \to 0$	M1	FT equivalent level of difficulty
	$dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 10$	A1	CAO. Must follow from correct working *Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0*
		5	
13	When x = 4, finding y = 0	B1	
	dy/dx = 10x – 20	M1	Ignore notation, e.g. y = 10x - 20, provided clear not from any wrong method.
	(when x = 4) gradient is 20	A1	
	Use of $\frac{y - y_1}{x - x_1} = m$ or y = mx + c	M1	Method to form equation FT their y value but not y=4, and their derived gradient
	y – 0 = 20 (x-4) or 0 = 20×4 + c, c = -80	A1	
	20x – y – 80 = 0	A1	CAO. Must be in this form, accept equivalents written as 3 terms with whole number coefficients with '=0' or '0='
		6	

	Additional Mathematics Summer 2017		Final Version
14	$\frac{36}{2x+5} + \frac{55}{3x-1}$ $36(3x - 1) + 55(2x + 5)$ as a numerator $(2x + 5)(3x - 1)$ as a denominator $\frac{218x + 239}{(2x + 5)(3x - 1)}$	M2 A1 A1 A1 5	M1 for either fraction, or M1 for sight of $36 \div (2x + 5)$ or $55 \div (3x - 1)$, with or without brackets FT provided M1 awarded due to a slip in the second fraction FT provided M1 awarded due to a slip in the second fraction CAO. Mark final answer If the denominator is expanded it must be correct If no marks, award SC2 for an answer of $\frac{218x + 239}{1980}$ (from starting with inverted fractions)
15	(a) General sin curve intersecting x-axis only at (0°,0), (180°,0) and (360°,0) Correct curve with 4 and -4 on y-axis (b) 14.477…(°) and 165.522…(°) only	M1 A1 B2 4	Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn at 90° and 270° in particular Must show a clear curve, not straight at turning points Accept rounded or truncated, e.g. 14(°) with 166(°) or 15(°) with 165(°) These values need to be selected, not amongst others unless unambiguously indicated as the response. B1 for sight of 14.477…(°) or 165.522…(°) B0 for a pair of angles with sum 180°
16	(a) $(60)x^{16/8}/x^{1/5}$ or equivalent first stage of work evaluated correctly with simplification of indices $60x^{9/5}$ (b) **Correctly** extracting a factor of $x^{1/5}$ or $x^{2/5}$ (to give correct numerator) OR correct alternative method with one **correct** step towards simplification $2x^{1/5} + x^{3/5}$	B1 B1 M1 A1 4	 CAO, must be simplified, allowing $60x^{1\frac{4}{5}}$ Mark final answer For an alternative method award M1 for $2x^{1/5} + \ldots$ or $\ldots + x^{3/5}$ CAO or equivalent factorised form. Mark final answer
17	(a) 225 (b) $\frac{1}{8+\sqrt{5}} \times \frac{8-\sqrt{5}}{8-\sqrt{5}}$ $= \frac{8-\sqrt{5}}{59}$	B1 M1 A1 3	*No marks if no working.* **Must** see $15^{1/3 \times 6}$ or 15^2 or $3^2 \times 5^2$ or 9×25 *No marks if no working.* Mark final answer

Differentiating from first principles. Marking guide.
Q12.

12	$y+\delta y = (x+\delta x)^2 + 10(x+\delta x)$ Intention to subtract (y=) $x^2 + 10x$ to find δy $\delta y = 2x\delta x + (\delta x)^2 + 10\delta x$ Dividing by δx and (lim) $\delta x \to 0$ $dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 10$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0

B1 For sight of $(x+\delta x)^2 + 10(x+\delta x)$ or $(x+h)^2 + 10(x+h)$ or using alternative notation.
This mark is given whether $(x+\delta x)^2 + 10(x+\delta x)$ stands alone or is embedded in an expression or a formula.

M1 For the <u>intent</u> to subtract $x^2 +10x$ from the above.
So $(x+\delta x)^2 + 10(x+\delta x) - x^2 +10x$ will gain the M1 even though there are missing brackets.
It can also be awarded to those who have expanded $(x+\delta x)^2 +10(x+\delta x)$ and then crossed out the x^2 term and the $+10x$ term.
Those who reverse the subtraction will gain M0 <u>unless</u> there is evidence later on of dividing by $-\delta x$.

A1 For sight of $2x\delta x + (\delta x)^2 +10\delta x$ (Accept δx^2 as meaning $(\delta x)^2$) with no other terms. Treat as a CAO.
$2x + \delta x +10$ will imply the above if division by δx has already been done.

M1 A FT, if of equivalent difficulty, is possible for this M1 (but not the subsequent A1).
A <u>correct</u> division by δx has to be done
(so if a FT it has to be correct for their $2x\delta x + (\delta x)^2 +10\delta x$)
<u>AND</u> we must see '$\lim \delta x \to 0$' OR '$\delta x \to 0$' OR 'δx tends to 0'.
It is M0 for '$\delta x = 0$' OR '$\delta x \approx 0$' OR 'δx is so small we can forget about it'.

All of the above marks can be gained even if there is no l.h.s. shown.

Final A1. Must be for a 'text book' quality presentation. E.g.
Has to be a correct l.h.s. for each line, 'δy' or '$\delta y/\delta x$'
AND at some point '$dy/dx = \lim_{\delta x \to 0} \delta y/\delta x$' or '$dy/dx = \lim_{\delta x \to 0} 2x + \delta x +10$'