

2.3 Higher tier

Foundation tier content is in standard text.

Intermediate tier content which is in addition to foundation tier content is in underlined text.

Higher tier content which is in addition to intermediate tier content is in **bold** text.

Higher tier – Number
<i>Understanding number and place value</i>
Reading and writing whole numbers of any magnitude expressed in figures or words. Rounding whole numbers to the nearest 10, 100, 1000, etc. Understanding place value and decimal places. Rounding decimals to the nearest whole number or a given number of decimal places. <u>Rounding numbers to a given number of significant figures.</u>
Using the equivalences between decimals, fractions, ratios and percentages. Converting numbers from one form into another. Ordering and comparing whole numbers, decimals, fractions and percentages. Understanding and using directed numbers, including ordering directed numbers.
<i>Understanding number relationships and methods of calculation</i>
Using the common properties of numbers, including odd, even, multiples, factors, primes. Expressing numbers as the product of their prime factors. <u>Least common multiple and highest common factor.</u> <u>Finding the LCM and HCF of numbers written as the product of their prime factors.</u> Using the terms square, square root, cube, <u>cube root and reciprocal.</u> The use of index notation for <u>zero</u> , positive <u>and negative</u> integral indices. <u>The use of index notation for positive unit fractional and other fractional indices.</u> Interpreting numbers written in standard form in the context of a calculator display. <u>Writing whole numbers in index form.</u> <u>Using the rules of indices.</u> <u>Expressing and using numbers in standard form with positive and negative powers of 10.</u>

Higher tier – Number

Using the facilities of a calculator, including the constant function, memory and brackets, to plan a calculation and evaluate expressions.

Using addition, subtraction, multiplication, division, square, square root, power, root, constant, memory, brackets and appropriate statistical functions.

Knowing how a calculator orders its operations. (Candidates will not be expected to list the key depressions that they have made.)

Using calculators effectively and efficiently.

Reading a calculator display correct to a specified number of decimal places or significant figures.

Using appropriate trigonometric functions on a calculator.

Understanding and using number operations and the relationships between them, including inverse operations and the hierarchy of operations.

Addition, subtraction, multiplication and division of whole numbers, decimals, fractions and negative numbers.

Finding a fraction or percentage of a quantity.

Expressing one number as a fraction or percentage of another.

Calculating fractional and percentage changes (increase and decrease), including the use of multipliers.

Repeated proportional changes; appreciation and depreciation.

Calculating using ratios in a variety of situations; proportional division.

Direct and inverse proportion.

The use of a non-calculator method to multiply and divide whole numbers up to and including the case of multiplication and division of a three-digit number by a two-digit number.

Estimating and approximating solutions to numerical calculations.

Using estimation in multiplication and division problems with whole numbers to obtain approximate answers, e.g. by first rounding the numbers involved to 1 significant figure.

Candidates must show sufficient working in order to demonstrate how they have obtained their estimate.

Recognising that recurring decimals are exact fractions, and that some exact fractions are recurring decimals.

Converting recurring decimals to fractional form.

Distinguishing between rational and irrational numbers.

Manipulating surds; using surds and π in exact calculations.

Higher tier – Number
<p>Simplifying numerical expressions involving surds, excluding the rationalisation of the denominator of a fraction such as $\frac{1}{(2-\sqrt{3})}$.</p>
<p><i>Solving numerical problems</i></p> <p>Interpretation and use of mathematical information presented in written or visual form when solving problems, e.g. TV programme schedules, bus/rail timetables, distance charts, holiday booking information.</p> <p>Money: The basic principles of personal and household finance, including fuel and other bills, hire purchase, discount, VAT, taxation, best buys, wages and salaries, loan repayments, mortgages, budgeting, exchange rates and commissions.</p> <p><u>Simple and compound interest, including the use of efficient calculation methods.</u></p> <p>Profit and loss.</p> <p><u>Finding the original quantity given the result of a proportional change.</u></p> <p>Foreign currencies and exchange rates.</p> <p>Carrying out calculations relating to enterprise, saving and borrowing, investing, appreciation and depreciation and understanding annual rates, e.g. AER, APR.</p>
<p>Giving solutions in the context of a problem, <u>selecting an appropriate degree of accuracy, interpreting the display on a calculator, and recognising limitations on the accuracy of data and measurements.</u></p> <p><u>Rounding an answer to a reasonable degree of accuracy in the light of the context.</u> Interpreting the display on a calculator. Knowing whether to round up or down as appropriate.</p> <p><u>Recognising that measurement is approximate and that a measurement expressed to a given unit is in possible error of half a unit.</u> <u>The upper and lower bounds of numbers expressed to a given degree of accuracy.</u></p> <p><u>Calculating the upper and lower bounds in the addition and subtraction of numbers expressed to a given degree of accuracy.</u></p> <p>Calculating the upper and lower bounds in calculations involving multiplication and division of numbers expressed to given degrees of accuracy.</p>
<p>Understanding and using Venn diagrams to solve problems.</p>

Higher tier – Algebra
<i>Understanding and using functional relationships</i>
Recognition, description and continuation of patterns in number. Description, in words <u>and symbols</u> , of the rule for the next term of a sequence.
Construction and interpretation of conversion graphs. Construction and interpretation of travel graphs. Construction and interpretation of graphs that describe real-life situations. Interpretation of graphical representation used in the media, recognising that some graphs may be misleading. <u>Recognising and interpreting graphs that illustrate direct and inverse proportion.</u>
Using coordinates in 4 quadrants. Drawing, interpreting, <u>recognising and sketching</u> the graphs of $x = a$, $y = b$, $y = ax + b$. <u>The gradients of parallel lines.</u>
Constructing and using tangents to curves to estimate rates of change for non-linear functions, and using appropriate compound measures to express results, including finding velocity in distance-time graphs and acceleration in velocity-time graphs. Interpreting the meaning of the area under a graph, including the area under velocity-time graphs and graphs in other practical and financial contexts. Using the trapezium rule to estimate the area under a curve.
<i>Understanding and using equations and formulae</i>
Substitution of positive and negative whole numbers, fractions and decimals into simple formulae expressed in words or in symbols. Understanding the basic conventions of algebra. Formation and simplification of expressions involving sums, differences, products <u>and powers</u> . Collection of like terms. Expansion of $a(bx + c)$, where a , b and c are integers. Formation, manipulation and solution of linear equations. <u>Changing the subject of a formula when the subject appears in one term.</u>
<u>The solution of linear equations with whole number coefficients in solving problems set in real-life contexts.</u>

Higher tier – Geometry and Measure

Understanding and using properties of shape

The geometrical terms: point, line, plane, parallel, right angle, clockwise and anticlockwise turns, perpendicular, horizontal, vertical, acute, obtuse and reflex angles, face, edge and vertex.

Vocabulary of triangles, quadrilaterals and circles: isosceles, equilateral, scalene, exterior/interior angle, diagonal, square, rectangle, parallelogram, rhombus, kite, trapezium, polygon, pentagon, hexagon, radius, diameter, tangent, circumference, chord, arc, sector, segment.

Simple solid figures: cube, cuboid, cylinder, prism, pyramid, cone, sphere, tetrahedron.

Interpretation and drawing of nets.

Using and drawing 2-D representations of 3-D shapes, including the use of isometric paper.

Accurate use of ruler, pair of compasses and protractor. (Lengths accurate to 2mm and angles accurate to 2° .)

Bisecting a given line, bisecting a given angle.

Constructing the perpendicular from a point to a line.

Essential properties of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus; classify quadrilaterals by their geometric properties.

Constructing 2-D shapes from given information and drawing plans and elevations of any 3-D solid.

Angles at a point. Angles at a point on a straight line.
Opposite angles at a vertex.

Parallel lines.
Corresponding, alternate and interior angles.

Angle properties of triangles.
Using the fact that the angle sum of a triangle is 180° .

Using Pythagoras' theorem in 2-D and 3-D, including reverse problems.

Using trigonometric relationships in right-angled triangles to solve problems, including those involving bearings and angles of elevation and depression.

Calculating a side or an angle of a right-angled triangle in 2-D and 3-D.

Higher tier – Geometry and Measure

Extending trigonometry to angles of any size.

The application of trigonometric functions to the solution of problems in 2-D or 3-D, including appropriate use of the sine and cosine rules.

Using the formula: area of a triangle = $\frac{1}{2}ab\sin C$.

Understanding and using properties of position, movement and transformation

Using the knowledge that, for two similar 2-D or 3-D shapes, one is an enlargement of the other.

Using the knowledge that, in similar shapes, corresponding dimensions are in the same ratio.

Using the relationships between the ratios of:

- lengths and areas of similar 2-D shapes, and
- lengths, areas and volumes of similar 3-D shapes.

Solving problems in the context of tiling patterns and tessellation.

Using and interpreting maps.

Interpretation and construction of scale drawings.

Scales may be written in the form 1 cm represents 5 m, or 1:500.

Use of bearings. (Three figure bearings will be used e.g. 065°, 237°.)

Constructing the locus of a point which moves such that it satisfies certain conditions, for example,

(i) a given distance from a fixed point or line,

(ii) equidistant from two fixed points or lines.

Solving problems involving intersecting loci in two dimensions.

Questions on loci may involve inequalities.

Higher tier – Geometry and Measure

Understanding and using measures

Standard metric units of length, mass and capacity.

The standard units of time; the 12- and 24- hour clock.
(The notation for the 12- and 24- hour clock will be 1:30 p.m. and 13:30.)

Knowledge and use of the relationship between metric units of length, mass, capacity, area and volume.

Making sensible estimates of measurements in everyday situations, recognising the appropriateness of units in different contexts.

Conversion between the following metric and Imperial units:
km - miles; cm, m - inches, feet; kg - lb; litres - pints, gallons.

Candidates will be expected to know the following approximate equivalences:
8km \approx 5 miles, 1kg \approx 2.2 lb, 1 litre \approx 1.75 pints

Reading and interpreting scales, including decimal scales.

Distinguishing between formulae for length, area and volume by considering dimensions.

Using compound measures including speed, density and population density.
Using compound measures such as m/s, km/h, mph, mpg, kg/m³, g/cm³, population per km²

Estimating the area of an irregular shape drawn on a square grid.

Calculating:

- perimeter and area of a square, rectangle, triangle, parallelogram, trapezium, circle, semicircle and composite shapes.

- surface area, cross-sectional area and volume of cubes, cuboids, prisms, cylinders and composite solids.

Lengths of circular arcs.

Perimeters and areas of sectors and segments of circles.

Surface areas and volumes of spheres, cones, pyramids and compound solids.

Higher tier – Statistics
<p>Understanding and using the statistical problem solving process: specifying the problem/planning; collecting, processing and representing data; interpreting and discussing results.</p>
<p><i>Specifying the problem and planning</i></p> <p>Specifying and testing hypotheses, taking account of the limitations of the data available.</p> <p><u>Testing an hypothesis such as ‘Girls tend to do better than boys in biology tests’.</u></p> <p><u>Specifying the data needed and considering potential sampling methods.</u> <u>Sampling systematically.</u></p> <p>Working with stratified sampling techniques and defining a random sample.</p> <p>Designing and criticising questions for a questionnaire, including notions of fairness and bias.</p> <p><u>Considering the effect of sample size and other factors that affect the reliability of conclusions drawn.</u></p>
<p><i>Processing, representing and interpreting data</i></p> <p>Sorting, classification and tabulation of qualitative (categorical) data, discrete or continuous quantitative data.</p> <p>Grouping of discrete or continuous data into class intervals of equal <u>or unequal</u> widths.</p> <p>Understanding and using tallying methods.</p> <p>Constructing and interpreting pictograms, bar charts and pie charts for qualitative data.</p> <p>Constructing and interpreting vertical line diagrams for discrete data.</p> <p>Constructing line graphs for the values of a variable at different points in time; understanding that intermediate values in a line graph may or may not have meaning.</p> <p>Constructing and interpreting grouped frequency diagrams <u>and frequency polygons.</u></p> <p>Temperature charts.</p> <p>Constructing and interpreting scatter diagrams for data on paired variables.</p> <p><u>Constructing and interpreting cumulative frequency tables and diagrams using the upper boundaries of the class intervals.</u></p>

Higher tier – Statistics

Extending skills in handling data into constructing and interpreting histograms with unequal class widths.

Frequency density.

Interpreting shapes of histograms representing distributions (with reference to mean and dispersion).

Selecting and using an appropriate measure of central tendency.

Mean, median and mode for a discrete (ungrouped) frequency distribution.

Estimates for the median and mean of grouped frequency distributions.

Comparison of two distributions using one measure of central tendency (i.e. the mean or the median) and/or one measure of spread.

Modal category for qualitative data.

Modal class for grouped data.

Estimating the median from a cumulative frequency diagram.

Selecting and calculating or estimating appropriate measures of spread, including the range and interquartile range applied to discrete, grouped and continuous data.

Producing and using box-and-whisker plots to compare distributions.

Drawing 'by eye' a line of 'best fit' on a scatter diagram, understanding and interpreting what this represents. [In questions where the mean point has been given, calculated or plotted, candidates will be expected to draw the line of 'best fit' through that point.]

Discussing results

Recognising that graphs may be misleading.
Looking at data to find patterns and exceptions.

Drawing inferences and conclusions from summary measures and data representations, relating results back to the original problem.

Drawing of conclusions from scatter diagrams; using terms such as positive correlation, negative correlation, little or no correlation.

Appreciating that correlation does not imply causality.

Estimating and calculating the probabilities of events

Understanding and using the vocabulary of probability, including notions of uncertainty and risk.

The terms 'fair', 'evens', 'certain', 'likely', 'unlikely' and 'impossible'.