LEVEL 2 MARKING SCHEME

SUMMER 2023

LEVEL 2
ADDITIONAL MATHEMATICS
9550-01

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

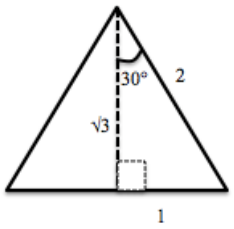
LEVEL 2 ADDITIONAL MATHEMATICS

SUMMER 2023 MARK SCHEME

		Mark	Comment
1			*Penalise including '+c' -1 only throughout*
	(a) $27x^8$ (+) $-8x^{-9}$ (+0)	B3	B1 for $27x^8$ (not $9\times3x^8$), B1 for $-8x^{-9}$ and B1 for +0 (or blank) provided at least one other mark awarded. If B3 penalise further incorrect working -1, e.g. treat further incorrect work with term $-8x^{-9}$ as ISW unless B3
	(b) $\frac{5}{6}x^{-\frac{1}{6}}$ or equivalent	B1	Index needs to be simplified. ISW
	(c) $\frac{-7x^{-8}}{4}$ or $\frac{-7}{4x^8}$	B1	CAO. ISW
		5	

2	$x^2 = 20x - 28$ written correctly showing correct completing of the square, e.g., for sight of any one of the following: • $(x – 10)^2 – 100 = -28$ • $(x – 10)^2 – 100 + 28 = 0$ • $100 – (x – 10)^2 = 28$ • $100 – (x – 10)^2 – 28 = 0$ • $(x – 10)^2 = 100 – 28$ • $– (x – 10)^2 + 100 = 28$ • $– (x – 10)^2 + 100 – 28 = 0$ • $– (x – 10)^2 = -100 + 28$ • $(x – 10)^2 = 72$ • $(x – 10)^2 – 72 = 0$ • $– (x – 10)^2 = -72$ • $– (x – 10)^2 + 72 = 0$ • $– (-x + 10)^2 + 100 – 28 = 0$ • $– (-x + 10)^2 + 72 = 0$ or equivalent $(x =) 10 \pm 6\sqrt{2}$ or $(x =) 10 + 6\sqrt{2}$ with $10 – 6\sqrt{2}$	M2 A2	*No working, no marks* '=0' may be implied in previous working, e.g., from sight of $x^2 – 20x + 28 (= 0)$, or further working M1 for sight of any one of the following, including if embedded in incorrect working: • $(x – 10)^2$ $(– 100)$ • $– (x – 10)^2$ $(+ 100)$ • $– (-x + 10)^2$ $(+ 100)$ OR Allow M1 for any one of the following, if not corrected in further correct working (which may then be M2): • $(x – 10\mathbf{x})^2 – 100 + 28$ • $– (x – 10\mathbf{x})^2 + 100 – 28$ • $(x +10)^2 – 100 + 28$ ISW for award of A2 or A1 Allow A2 for strict FT from M1 **and**: • $(x – 10)^2 = 100 + 28$ for an answer of $10 \pm 8\sqrt{2}$ • $(x + 10)^2 = 100 – 28$ for an answer of $-10 \pm 6\sqrt{2}$ A1 for any of the following, (x =) • $10 \pm 2\sqrt{18}$ • $10 \pm 3\sqrt{8}$ • $10 + 6\sqrt{2}$ • $10 – 6\sqrt{2}$ OR A1 for strict FT from M1 and $(x – 10)^2 = 100 + 28$, for: • $10 \pm 2\sqrt{32}$ • $10 \pm 4\sqrt{8}$ • $10 + 8\sqrt{2}$ • $10 – 8\sqrt{2}$ OR A1 for strict FT from M1 and $(x + 10)^2 = 100 – 28$, for: • $-10 \pm 2\sqrt{18}$ • $-10 \pm 3\sqrt{8}$ • $-10 + 6\sqrt{2}$ • $-10 – 6\sqrt{2}$ If no marks, Award SC1 for either: • $-10 \pm 8\sqrt{2}$ from $(x + 10)^2 = 128$ • 'their simplified $a \pm b\sqrt{c}$' from their $(x ….)^2 = …$ provided equivalent level of difficulty with $a > 0$, $c>0$ and b is a product of at least 2 integers No FT for A mark when attempting to write the square root of a negative number, e.g. $\sqrt{-72}$ or $\sqrt{-128}$
		4	If no marks, award SC1 for $\sqrt{72} = \pm 6\sqrt{2}$ or $\sqrt{128} = \pm 8\sqrt{2}$

3	(a) $12x^{7/10}$ (b) x^5 (c) $2 + 5x^{½}$ or $2 + 5\sqrt{x}$	 B1 B1 B2 4	Indices must be simplified throughout Must be simplified Allow $1(2 + 5x^{½})$ For award of B2 mark final answer B1 for any of the following: • …. + $5x^{½}$ • …. +$5\sqrt{x}$ • 2 + …. • $2(x^0) + 5x^{2/4}$
4	$x^2 + 5x – 36 = 0$ or $0.5x^2 + 2.5x – 18 = 0$ or equivalent $(x – 4)(x + 9)$ (= 0) or $(x/2 – 2)(x + 9)$ (= 0) x = 4 with x = -9	B2 M1 A1 4	Must show or imply in further working '=0' B1 for any one of the following: • $x^2 + 5x = 18 \times 2$ • $x^2/2 + 2.5x = 18$ M0 A0 if factorising not seen Factorisation must be seen FT their $ax^2 + bx + c = 0$ provided it factorises and $a\neq0$, $b\neq0$ and $c\neq0$

5	(a) (Width =) $\frac{142}{2 + 5\sqrt{3}}$ or $142 \div (2 + 5\sqrt{3})$	B2	B1 for width × $(2 + 5\sqrt{3}) = 142$ or $a(b + c\sqrt{d}) \times (2 + 5\sqrt{3}) = 142$ Allow B1 for $x \times 2 + 5\sqrt{3} = 142$ or $142 \div 2 + 5\sqrt{3}$ unless treated correctly in further working
	$\frac{142}{2 + 5\sqrt{3}} \times \frac{2 - 5\sqrt{3}}{2 - 5\sqrt{3}}$ or $\frac{142(2 - 5\sqrt{3})}{4 - 75}$ or $-2(2 - 5\sqrt{3})$	M1	Working MUST be seen. No working, no marks Depends on B2 previously awarded
	$2(5\sqrt{3} - 2)$ or $2(-2 + 5\sqrt{3})$	A1	CAO. Mark final answer
	(b) Annotated diagram showing: • perpendicular (right angle indicated) • (unambiguous) angle 30° (and right angle) • (length of ½ side as 1) • with √3 derived or correct indication For example:  	B2 6	Allow diagram with an appropriate height drawn without the right angle indicated as 'perpendicular' and a 'right angle' Allow if the correct angle of 30° is indicated unambiguously but not labelled Do not accept if both the 30° and 90° angles are both indicated by single arcs without labels Allow if length ½ side as 1 is omitted B1 for sight of one of the following: • (perpendicular) height **and** (unambiguous) angle 30° • √3 derived or correct indication
6	(a) Substitution of x = -5, $4(-5)^3 - 2(-5)^2 - (-5)$ (= –500 – 50 + 5) -545	M1 A1	Or division method giving $4x^2 - 22x$ .... Allow for intention, e.g. $4(-5)^3 - 2(-5)^2$ **- 5** (= -555) Mark final answer
	(b)(i) Substitute x = 2, $(2)^3 - 6(2)^2 - 13(2) + 42$ (= 8 - 24 - 26 + 42) Showing f(2) = 0	M1 A1	Or division method giving $x^2 - 4x$ … Accept sight of **substitution** with '=0' shown
	(ii) $(x-2)(x^2 + bx + c)$ or intention to divide by (x - 2) with x^2 shown	M1	If any values are inserted at least 1 needs to be correct. Appropriate sight of -4x or -21 implies M1 (and possible A1 to follow)
	((x-2)) ($x^2 - 4x - 21$)	A2	A1 for -4x or -21 Or use of factor theorem A1 (x + 3), A1 (x - 7)
	(x - 2)(x + 3)(x - 7)	A1 8	CAO, with all 3 factors shown, ignore sight of "=0", ISW **Must** be from sight of $x^2 - 4x - 21$ previously No working to show factorising is M0 A0 A0

7	(Surface area) $326.4 = \pi \times 5.6\,(5.6 + l)$ or $326.4 = \pi \times 5.6^2 + \pi \times 5.6 \times l$	M1	
	$l = \frac{326.4}{\pi \times 5.6} - 5.6$ or $\frac{326.4 - 5.6^2 \times \pi}{\pi \times 5.6}$	m1	(= 12.95.... cm or 12.96… or 13 cm) Allow l = 12.95, 12.96 or 13 from trial & improvement
	$(h^2 =)\ (12.95....)^2 - 5.6^2$	M1	FT 'their derived l'
	$h^2 = 136.418...$ or $(h =) \sqrt{136.418...}$	A1	FT 'their derived l' provided `> 5.6
	(Vertical height) h = 11.6798... (cm)	A1	Allow rounding or truncation of h FT M1 A0 to allow A1 for square root of 'their 136.418... provided leads to 'their h' < 'their l' *Note:* *l* / h^2 / *h*: *12.95* / *136.34…* / *11.676..*; *12.96* / *136.60…* / *11.687…*; *12.9* / *135.05* / *11.62…*; *13* / *137.64* / *11.73…*
	(Volume V =) $\frac{1}{3} \times \pi \times 5.6^2 \times 11.6798...$	M1	FT 'their h' > 0, provided at least M1 previously awarded Not for use of 'their l'
	Answers in the range 383.2 (cm^3) to 385.3 (cm^3)	A1	CAO, not a FT Do not FT from allowance of premature rounding or truncation of h for previous A1 as 'their answer' will now be outside the range allowed
	QWC2: • Candidates will be expected to present work clearly, with words explaining process or steps AND • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer QWC1: Candidates will be expected to • present work clearly, with words explaining process or steps OR • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer	QWC 2 9	QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar. QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar. QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.

8	$\frac{10x + 3}{2} = 4x^2 + 2x - 3$ or $10x - 2(4x^2 + 2x - 3) + 3 = 0$ or equivalent	M1	For M1 allow intention, with no more than 1 slip or for use of $y = 5x + 3$
	$8x^2 - 6x - 9 = 0$ or equivalent	A1	CAO. Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0
	$x = \frac{-(-6) \pm \sqrt{((-6)^2 - 4\times8\times-9)}}{2\times8}$ or (2x ...3) (4x ...3) or equivalent	m1	Working **must** be seen for m1 to be awarded If not solved by factorising, the use of correct quadratic formula must be seen, allow 1 slip in substitution (not a slip with the formula) FT provided M1awarded for equivalent level of difficulty
	$x = \frac{6 \pm \sqrt{324}}{16}$ or (2x - 3) (4x + 3) (=0)	A1	Or equivalent Do not FT from m1 awarded when there has been a slip in substitution
	$x = 1.5$ **with** $x = -0.75$	A1	Or equivalent. ISW
	$x = 1.5$ **and** $y = 9$ **with** $x = -0.75$ **and** $y = -2.25$	A1	Or equivalent FT provided M1, m1 previously awarded using their values of x in $(10x + 3)/2$ or $4x^2 + 2x - 3$, but not in an incorrect rearrangement of the given equations, to find y-values Accept answers given as coordinates
	Alternative using $x = (2y - 3)/10$		
	$y = 4\frac{(2y - 3)^2}{10^2} + 2\frac{(2y - 3)}{10} - 3$ or equivalent	*M1*	
	$16y^2 - 108y - 324 = 0$ or $4y^2 - 27y - 81 = 0$ or equivalent	*A1*	*Must equate to zero*
	$y = \{27 \pm \sqrt{(27^2 - 4\times4\times-81)}\}/2\times4$ or (4y...9)(y...9) or equivalent	*m1*	*Working **must** be seen for m1 to be awarded If not solved by factorising, the use of correct quadratic formula must be seen, allow 1 slip in substitution (not a slip with the formula) FT provided M1awarded for equivalent level of difficulty*
	$y = (27 \pm \sqrt{2025})/8$ or (4y + 9)(y - 9) or equivalent	*A1*	*Do not FT from m1 if given when including a slip in substitution*
	*$y = -2.25$ **with** $y = 9$* *$x = 1.5$ **and** $y = 9$ **with** $x = -0.75$ **and** $y = -2.25$*	*A1* *A1*	*Or equivalent. ISW* *Or equivalent FT to final A1, provided M1, m1 previously awarded using their values of y in $(2y - 3)/10$* or *$y = 4x^2 + 2x - 3$, but not in an incorrect rearrangement of the given equations, to find x-values*
		6	

9	$264x^{10}$	B2 2	B1 for sight of $24x^{11}$. FT to 2nd B1 from $dy/dx = kx^n$ Ignore incorrect notation
10	(a)(i) $(AB^2 =)\ (4 - -8)^2 + (6 - 1)^2$ $(=12^2 + 5^2)$ or $AB = \sqrt{169}$ $(AB =)\ 13$ (a)(ii) Gradient AB $(6 - 1) / (4 - - 8)$ $= 5/12$ Perpendicular gradient is $- 12/5$ (a)(iii) $(4 + - 8)/2, (6 + 1)/2$ or equivalent Midpoint AB (-2, 3.5) or equivalent (b) $9 = 4\times-3 + c$ or $y - 9 = 4(x - -3)$ or $4 = \frac{y - 9}{x - -3}$ $y = 4x + 21$	M1 A1 M1 A1 B1 M1 A1 M1 A1 9	Or equivalent CAO. An unsupported 13 is awarded M1 A1 Check if 4 - -8 and 6-1 are from incorrect evaluations in (a)(i), provided these calculations are seen, allow M1 but A0 Mark final answer and then FT Must be simplified if FT is a whole number Allow -5/-12 provided not contradicted as -5/12 in further working Allow 0.416(66…) For M1 A1, but FT for B1 must be given as -2.4 FT -1/'their 5/12' Award of B1 for -12/5 implies previous M1 and A1 CAO. ISW. Allow (x =) -2 (and y =) 3.5 Award M1 A1 for (-2, 3.5) if unsupported, provided not from incorrect working CAO. Must be in this form
11	(a) $14x^7/7 - 5x + 4x^{-2}/-2$ $2x^7 - 5x - 2x^{-2}$ or $2x^7 - 5x - 2/x^2$ $+ c$ (constant) (b) $8x^4/4 + 4x^2/2$ or $2x^4 + 2x^2$ Use of correct limits 3 & 2 in correct order **and** intention to subtract $((2\times81 + 2\times9) - (2\times16 + 2\times4) =)$ 140	B3 B1 B1 M2 m1 A1 9	B1 for each term ISW from correct unsimplified form. CAO simplified form. Mark final answer Awarded only if at least B1 is awarded for integration *No workings, no marks* Ignore sight of '+c' for M marks only M1 one term correct. CAO. Must be from correct working

12	(Diagonal^2 =) $3^2 + 3^2$ (Diagonal =) $\sqrt{18}$ or $3\sqrt{2}$ or 4.24...(cm)	M1 A1	Or ((½ diagonal)^2 =) $(3^2 + 3^2) \div 2$ Or (½ diagonal =) $\frac{\sqrt{18}}{2}$ or $\frac{3\sqrt{2}}{2}$ or 2.12...(cm)
	(Angle x =) $\cos^{-1} \frac{½\sqrt{18}}{6}$ or $\cos^{-1} \frac{6^2 + (\sqrt{18})^2 - 6^2}{2\times6\times\sqrt{18}}$ (= $\cos^{-1}$ 0.35355...)	M2	FT 'their derived diagonal' or 'their derived ½diagonal' M1 for $6^2 = 6^2 + (\sqrt{18})^2 - 2\times6\times\sqrt{18}\times\cos x$ or cos x = $\frac{½\sqrt{18}}{6}$ or $\frac{\sqrt{18}}{12}$ or $\frac{\sqrt{2}}{4}$ *Note: If $3\sqrt{2}$ is used in the cosine rule, need $(3\sqrt{2})^2$ or correct intention implied by further working*
	An answer in the range 69.29(°) to 69.31(°)	A1	Allow an answer rounded or truncated to 69(°) provided it is from working that would lead an angle in the range 69.29(°) to 69.31(°)
	Alternative: *((½ diagonal)^2 =) $(3^2 + 3^2) \div 2$* *(½ diagonal =) $\frac{\sqrt{18}}{2}$ or $\frac{3\sqrt{2}}{2}$ or 2.12...(cm)*	*M1* *A1*	*Or (Diagonal^2 =) $3^2 + 3^2$* *Or (Diagonal =) $\sqrt{18}$ or $3\sqrt{2}$ or 4.24...(cm)*
	(Height^2 =) $6^2 - (\frac{\sqrt{18}}{2})^2$ *or (Height^2 =) $6^2 - 2.12..^2$ (= 31.5 cm)* *or (Height^2 =) $6^2 - (3/2)^2 - (3/2)^2$* *or (Height =) $\sqrt{31.5}$ (= 5.61…cm)*	*M1*	*FT 'their derived ½diagonal' or 'their derived diagonal'*
	sin x = 5.61…/ 6 (= 0.935..) *or* *tan x = 5.61… / 2.12 (= 2.646…)*	*m1*	*Depends on previous M1 awarded*
	An answer in the range 69.29(°) to 69.31(°)	*A1*	*Allow an answer rounded or truncated to 69(°) provided it is from working that would lead to an angle in the range 69.29(°) to 69.31(°)*
		5	

13	$(dy/dx=) \frac{3x^2}{3} + 2x - 15$ or $x^2 + 2x - 15$	B1	Must not be from sight of an incorrect equation, i.e. finding dy/dx from $y = x^3 + 3x^2 - 45x$ is awarded B0
	$dy/dx = 0$ or $x^2 + 2x - 15 = 0$	M1	FT their dy/dx from $ax^2 + bx$ throughout
	$x = \{-(2) \pm \sqrt{(2^2 - 4\times1\times-15)}\}/2$ or $(x - 3)(x + 5) = 0$	m1	If the quadratic formula used, working must be shown, allow 1 slip in substitution
	$x = 3$ and $x = -5$	A1	*y-coordinates not required*
	$d^2y/dx^2 = 2x + 2$	M1	*Method for determining min or max MUST be shown, final answer only is M0 here, then A0, A0* Or first derivative test, interpretation of first derivative test. Or alternative. FT 'their dy/dx' provided equivalent difficulty
	At $x = -5$, $d^2y/dx^2 < 0$, point is a maximum At $x = 3$, $d^2y/dx^2 > 0$, point is a minimum	A1 A1	FT for 'their x value' FT for 'their other x value' provided this does not have the same interpretation as the first x value *If M0A0A0, award SC1 for correct FT from 'their $d^2y/dx^2 = ax + b$, $a>0$' applied correctly provided it leads to 1 maximum and 1 minimum* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
		7	
14	$y+\delta y = 3(x+\delta x)^2 + 4$	B1	Or alternative notation.
	Intention to subtract $(y=) 3x^2 + 4$ to find δy	M1	
	$\delta y = 6x\delta x + 3(\delta x)^2$	A1	Accept δx^2 as meaning $(\delta x)^2$
	Dividing by δx **and** (lim) $\delta x \to 0$	M1	FT equivalent level of difficulty
	$dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 6x$	A1	CAO. Must follow from correct working *Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0*
		5	

15	(a) Sine curve that • intersects x-axis at (0°, 0) (90°, 0) (180°, 0) (270°, 0) and (360°, 0), • has maxima at ≈ 45° and 225° and minima at ≈ 135° and 315° • has 3 and -3 indicated on the y-axis	B2	Must show a clear curve, not straight at turning points B1 for one of the following, allowing straight rather than curves at turning points: • a sine curve intersecting x-axis at (0°, 0) (90°, 0) (180°, 0) (270°, 0) and (360°, 0) • a sine curve (which may have incorrect period) with 3 and -3 indicated on the y-axis
	(b) 7.5°, 82.5°, 187.5° and 262.5° alone	B2 4	Must be correct to 1 decimal place and not from incorrect working B1 for sight of 7.5(°) or for the correct 4 angles but not given to 1 decimal place Note: sin2x = 0.777/3 = 0.259, followed by incorrect working x = 0.259/sin2 = 7.42… is B0
16	Curve intersections with x-axis 3 and 9 or appropriate sight of x = 3 and x = 9	B2	B1 for any of the following • either intersection correct • sight of ○ -(x – 3)(x – 9) ○ (x – 3)(9 – x) ○ (3 – x)(x – 9) ○ (x – 3)(x – 9) ○ (-12 $\pm\sqrt{36}$)/-2 ○ (12 $\pm\sqrt{36}$)/2 or equivalent
	Intention to integrate	M1	Intention to integrate, hence not using given or differentiated expression
	$-x^3/3 + 12x^2/2 - 27x$	A2	Ignore sight of '+c. A1 one term correct
	Use of correct limits 9 & 3 in correct order **and** intention to subtract	m1	FT 'their stated value 9' > 0 and 'their stated value 3' > 0 in appropriate order provided 'their stated 9' ≠ 'their stated 3' There must be evidence of use of limits and subtraction
	36	A1 7	CAO Do not accept 36 + c *Correct answer only gets B2 M1 A0 m0 A0 (for intention to integrate)* *No marks for use of the trapezium rule*

17	When x = 2, finding y = 16	B1	
	dy/dx = 10x - 3	M1	
	when x = 2 gradient is 17	A1	Must be from sight of dy/dx = 10x - 3
	Use of $y - y_1 = m(x - x_1)$ or $y = mx + c$ or $m = \frac{y - y_1}{x - x_1}$	M1	Method to form equation with appropriate substitution for at least two of x, y and m. FT 'their y value' (but not y=2) and 'their derived gradient'. Needs to be x = 2, do not FT 'their x'
	$y - 16 = 17(x - 2)$ or $16 = 17\times2 + c$, $c = -18$ or $y = 17x - 18$	A1	FT for x = 2, 'their y' and 'their derived m'
	$17x - y - 18 = 0$ or $-17x + y + 18 = 0$	A1 6	CAO. Allow terms in other orders provided '= 0' Mark final answer