LEVEL 2 MARKING SCHEME

WINTER 2022

LEVEL 2
ADDITIONAL MATHEMATICS
9550-01

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

LEVEL 2 ADDITIONAL MATHEMATICS

WINTER 2022 MARK SCHEME

		Mark	Comment
1	(a) $30x^9 + 8x$ (+) $-4x^{-5}$ (+0)	B4	B1 for $30x^9$ (not $3\times10x^9$), B1 for (+)8x (not $2\times4x$), B1 for $-4x^{-5}$ and B1 for +0 (or blank) provided at least one other mark awarded. If B4 penalise further incorrect working -1, e.g. treat further incorrect work with term $-4x^{-5}$ as ISW unless B4
	(b) $4x^{-⅕}$ or equivalent	B1	Allow $5\times⅘x^{-⅕}$. Index needs to be simplified. ISW
	(c) $-16x^{-9}$ or $-16/x^9$	B1	CAO. Index and coefficient need to be simplified. ISW
		6	*Penalise including '+c' -1 only throughout*
2	$(2\sqrt{5} + 3 + 2\sqrt{5} - 3)(2\sqrt{5} + 3 - 2\sqrt{5} + 3)$ OR $((2\sqrt{5})^2 + 12\sqrt{5} + 9) - ((2\sqrt{5})^2 - 12\sqrt{5} + 9)$ OR $(20 + 12\sqrt{5} + 9) - (20 - 12\sqrt{5} + 9)$	M2	M1 if no more than 2 terms incorrect
	$4\sqrt{5} \times 6$ OR $12\sqrt{5} + 12\sqrt{5}$	A1	FT from M1 for equivalent level of difficulty
	$24\sqrt{5}$	A1	CAO
		4	
3	(a) 1/81	B2	*No marks if no working.* B1 for sight of 3^{-4} or $1/3^4$ Mark final answer
	(b) $\frac{1}{8 - \sqrt{5}} \times \frac{8 + \sqrt{5}}{8 + \sqrt{5}}$	M1	*No marks if no working.*
	$= \frac{8 + \sqrt{5}}{59}$	A1	Mark final answer
		4	
4	(a) $36x^{8/5}$	B1	ISW. Allow $36x^{13/5}$ or $36x^{1.6}$
	(b) $x^{½}$	B1	ISW
	(c) Extracting a factor of $(7)x^{1/9}$ OR correct alternative method with one correct step towards simplification	M1	At least 2 terms within the numerator brackets must be correct For an alternative method $\frac{7x^{2/9}}{7x^{1/9}} + \frac{14x^{1/9}}{7x^{1/9}} + \frac{7x^{5/9}}{7x^{1/9}}$ award M1 only when at least 2 of these 3 fractions have been simplified correctly
	$x^{1/9} + 2 + x^{4/9}$	A1	CAO. Do not accept $2x^0$ for 2. Mark final answer
		4	
5	(a) $4(-2)^3 - 2(-2)^2 + 10(-2) + 1$ $(= -32 - 8 - 20 + 1)$	M1	Or division method giving $4x^2 - 10x \ldots$
	-59	A1	
	(b)(i) Substitute x = -5 Showing f(-5) = 0	M1 A1	Or division method giving $x^2 + 6x \ldots$ Accept sight of substitution with '=0' shown
	(ii) $(x+5)(x^2 + bx + c)$ or intention to divide by $(x+5)$ with x^2 shown	M1	If any values are inserted at least 1 needs to be correct, appropriate sight of +6x or -16 implies M1 (and A1 to follow)
	$(x+5)$ $(x^2 + 6x - 16)$	A2	A1 for +6x or -16 Or use of factor theorem A1 $(x + 8)$, A1 $(x - 2)$
	$(x+5)(x+8)(x-2)$	A1	CAO, with all 3 factor shown, ignore sight of "=0", ISW Must be from sight of $x^2 + 6x - 16$ previously
		8	

6	(Area of right-angled triangle is) ½ (x + 2)(3x + 20) = 220 $3x^2 + 26x – 400 = 0$ (x – 8)(3x + 50) = 0 or $x = \frac{-26 \pm \sqrt{5476}}{6}$ or $x = \frac{-26 \pm 74}{6}$ x = 8 (cm) (x = -16.66(....) cm) **AND** (Lengths of two sides given are) (x + 2 =) 10 (cm) and (3x + 20=) 44 (cm) (Hypotenuse^2 =) $10^2 + 44^2$ (= 2036) (Hypotenuse is) 45(.122.... cm) (Perimeter 10 + 44 + 45.12... =) 99(.12....cm) QWC2: • Candidates will be expected to present work clearly, with words explaining process or steps AND • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer QWC1: Candidates will be expected to • present work clearly, with words explaining process or steps OR • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer	M1 A1 m1 A1 M1 A1 A1 QWC 2 9	Accept (x + 2)(3x + 20) = 2×220 FT for equivalent level of difficulty Allow for $x = \frac{-26 + \sqrt{5476}}{6}$ alone Ignore negative solution FT 'their derived lengths of two sides' FT 'their derived lengths of two sides' FT provided a unique answer is given and both previous M1 marks have been awarded *If no marks,* *award SC2 for (hypotenuse^2 =) $10x^2 + 124x + 404$* *or SC1 for (hypotenuse^2 =) $(x + 2)^2 + (3x + 20)^2$* QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar. QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar. QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.
7	$(x + 25)^2$ (± …) (Minimum value at x =) -25 (Minimum value is) (+) 75	M1 A1 A1 3	Ignore 'their (± …)' or '=0' Do not accept method dy/dx =2x+50 CAO CAO, from $(x + 25)^2 + 75$

8	$5x - 2 = 3x^2 + 4x - 6$ $3x^2 - x - 4 = 0$	M1 A1	Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0
	$x = \frac{-(-1) \pm\sqrt{((-1)^2 - 4\times3\times-4}}{2\times3}$ or $(3x \ldots4)(x \ldots1)$	m1	**Must show use** of correct quadratic formula, allow 1 slip in substitution (not a slip with the formula) If completing the square used award m1for sight of $3(x + -1/6)^2 \pm \ldots$
	$x = \frac{1 \pm\sqrt{49}}{6}$ or $(3x - 4)(x + 1)$ (=0)	A1	
	x = 4/3 or equivalent **and** x = -1	A1	Allow truncation to 1.3 from correct working
	x = 4/3 **and** y = 14/3 or equivalents **with** x = -1 **and** y = -7	A1	FT provided M1, m1 previously awarded using their values of x in 5x - 2 or equivalent to find y-values Accept answers given as coordinates *Alternative using x = (y + 2)/5* M1 $y = 3\frac{(y+2)^2}{5^2} + \frac{4(y+2)}{5} - 6$ or equivalent A1 $3y^2 + 7y - 98 = 0$ or equivalent (equate to zero) m1 $y = \{-7\pm\sqrt{(7^2 - 4\times3\times-98)}\}/2\times3$ or $(y \ldots7)(3y \ldots14)$ Allow 1 slip in substitution A1 $y = (-7\pm\sqrt{1225})/6$ or $(y + 7)(3y - 14)$ A1 y = 14/3 or equivalent **and** y = -7 A1 x = 4/3, y = 14/3 or equivalents **with** x = -1, y = -7 FT to final A1, provided M1, m1 previously awarded using their values of y in (y + 2)/5 or equivalent to find x-values
		6	
9	cos obtuse angle = $\frac{14.4^2 + 10.2^2 - 18.8^2}{2 \times 14.4 \times 10.2}$	M2	M1 for unrearranged form $18.8^2 = 14.4^2 + 10.2^2 - 2 \times 14.4 \times 10.2 \times$ cos obtuse angle
	(Obtuse angle =) 98(.2...°)	A1	
	(Vertical height =) 14.4 × sin (180 – 98(.2...)) 14.25.... (cm) or 14.3 (cm)	M1 A1	
	(Volume =) ⅓ × π × (10.2 ÷ 2)² × 14.25... Answer in the range 387.9 (cm^3) to 390 (cm^3)	M1 A1	*Alternative 1:* *Cos acute angle* = $\frac{18.8^2 + 10.2^2 - 14.4^2}{2 \times 18.8 \times 10.2}$ *(Acute angle =) 49(.29...°)* *(Vertical height =) 18.8 × sin 49(.29..)* *14.25.... (cm) or 14.3 (cm)* *(Volume =) ⅓ × π × (10.2 ÷ 2)² × 14.25...* *Answer in the range 387.9 (cm^3) to 390 (cm^3)* *Alternative 2:* *$h^2 + x^2 = 14.4^2$ AND $h^2 + (x + 10.2)^2 = 18.8^2$* *M1* *$(x + 10.2)^2 - x^2 = 18.8^2 - 14.4^2$* *m1* *$20.4x = 18.8^2 - 14.4^2 - 10.2^2$* *m1* *x = 2.06(07... cm)* *A1* *(Vertical height h =) 14.25.... (cm) or 14.3 (cm)* *B1* *(Volume =) ⅓ × π × (10.2 ÷ 2)² × 14.25...* *M1* *Answer in the range 387.9 (cm^3) to 390 (cm^3)* *A1*
		7	

10	(a) $60x^4$	B2	B1 for sight of $12x^5$. FT to 2nd B1 from $dy/dx = kx^n$ Ignore incorrect notation
	(b) For sight of $(dy/dx =) 3ax^2 + 2bx + c$ OR $(y =) \frac{15x^3}{3} + \frac{8x^2}{2} + 3x$ (+ constant)	B1	May be implied by 2 or 3 correct values
	$a = 5$ $b = 4$ $c = 3$ $d = 30$	B3 6	B2 for any 2 or 3 values correct, or B1 for 1 value correct *Accept sight of correct answers from 'uncorrected' working* *Only accept embedded answers if clearly stated unambiguously*
11	(a) $(AB^2 =) (10 - -6)^2 + (8 - 2)^2 (=16^2 + 6^2)$	M1	Or equivalent. Allow 1 slip in sign of substitution
	$AB = \sqrt{292}$ $= 2\sqrt{73}$	A1 B1	CAO. Allow for sight of 17(.088...) FT 'their AB' of equivalent difficulty expressed correctly, e.g. needs to be in the form $a\sqrt{b}$ where $a \neq 1$ and $b \neq 1$ or simpler Sight of $2\sqrt{73}$ implies previous $\sqrt{292}$
	(b) Gradient AB $(8 - 2) / (10 - -6)$ $= 6/16$ (= ⅜) Perpendicular gradient -16/6 or -8/3	M1 A1 B1	Or equivalent CAO. Mark final answer and then FT FT -1/'their gradient'
	$(10 + - 6)/2, (8 + 2)/2$ Midpoint AB (2, 5) or equivalent	M1 A1	Accept (2, …) or (… , 5) CAO
	Use of $y=mx+c$ or $y-y_1= m(x-x_1)$ or $m = \frac{y-y_1}{x-x_1}$	M1	Relevant substitutions must be seen OR for an alternative correct method of finding the equation of a straight line, for the idea of how an equation of a straight line can be found. FT 'their perpendicular gradient' AND 'their mid point' or for 'points A or B' used
	$8x + 3y – 31 = 0$ or equivalent unsimplified equation, e.g. • $3y = -8x + 31$ • $y = -8x/3 + 31/3$ • $3y – 15 = -8x + 16$ • $3(y – 5) = -8(x – 2)$	A1 10	CAO. Must have terms in x and y, not with x and y involved in a quotient. ISW Note: final answer of $\frac{y – 5}{x – 2} = -\frac{8}{3}$ is awarded final M1, A0
12	$14x^7/7 + 16x^4/4 – 2x + 4x^{-2}/-2$	B4	B1 for each term ISW from correct unsimplified form.
	$2x^7 + 4x^4 – 2x – 2x^{-2}$ or $2x^7 + 4x^4 – 2x – 2/x^2$ + c (constant)	B1 B1 6	CAO simplified form Awarded only if at least B1 is awarded for integration

13	(dy/dx=) $9x^2 - 18x$ dy/dx = 0 or $9x^2 - 18x = 0$ $x = 0$ and $y = 5$ $x = 2$ and $y = -7$ $d^2y/dx^2 = 18x - 18$ At (0, 5) $d^2y/dx^2 < 0$, point is a maximum At (2, -7): $d^2y/dx^2 > 0$, point is a minimum	B1 M1 A1 A1 M1 A1 A1 7	 FT their dy/dx from $ax^2 + bx$ throughout If A0, A0 here, award A1 for x = 0 with x = 2 *Answer only, no working shown M0 A0 A0* *Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0* Or first derivative test, interpretation of first derivative test. Or alternative. FT 'their dy/dx' for M1 provided equivalent difficulty FT for 'their x value' FT for 'their other x value' provided this does not have the same interpretation as the first x value *If M0A0A0, award SC1 for correct FT from 'their $d^2y/dx^2 = ax + b$, $a>0$' applied correctly provided it leads to 1 maximum and 1 minimum* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
14	$y+\delta y = (x+\delta x)^2 + 17(x+\delta x)$ Intention to subtract (y=) $x^2 + 17x$ to find δy $\delta y = 2x\delta x + (\delta x)^2 + 17\delta x$ Dividing by δx **and** (lim) $\delta x \to 0$ $dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 17$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working *Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0*
15	(a) General sine curve intersecting x-axis only at (0°,0) , (180°,0) and (360°,0) Correct curve with 5 and -5 on y-axis (b) 39(.05…°) **and** 140.9(...°) or 141° only	M1 A1 B2 4	Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn at 90°, and 270° in particular Must show a clear curve, not straight at turning points Accept rounded or truncated These values need to be selected, not amongst others unless unambiguously indicated as the response. B1 for sight of either angle
16	Intention to integrate $-x^3/3 + 4x^2/2 + 5x$ Use of correct limits 5 & 3 in correct order **and** intention to subtract 28/3 or equivalent	M1 A2 m1 A1 5	Intention to integrate, hence not using given or differentiated expression Ignore sight of '+C. A1 one term correct. CAO Allow 9.3 Do not accept 9⅓ + C *Correct answer only gets M1 A0 m0 A0 (for intention to integrate)* *No marks for use of the trapezium rule*

17	When $x = 2$, finding $y = 15$	B1	
	$dy/dx = 10x - 4$	M1	
	(when $x = 2$) gradient is 16	A1	Must be from sight of $dy/dx = 10x - 4$
	Use of $y - y_1 = m(x - x_1)$ or $y = mx + c$ or $m = \frac{y - y_1}{x - x_1}$	M1	Method to form equation with appropriate substitution for at least two of x, y and m. FT their y value (but not y=16) and their derived gradient
	$y - 15 = 16(x-2)$ or $15 = 16\times2 + c$, $c = -17$	A1	
	$y = 16x - 17$	A1	CAO. Mark final answer
		6	

Differentiating from first principles. Marking guide.

Q14.

14	$y+\delta y = (x+\delta x)^2 + 17(x+\delta x)$ Intention to subtract (y=) $x^2 + 17x$ to find δy $\delta y = 2x\delta x + (\delta x)^2 + 17\delta x$ Dividing by δx and (lim) $\delta x \to 0$ $dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 17$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0

B1 For sight of $(x+\delta x)^2 + 17(x+\delta x)$ or $(x+h)^2 + 17(x+h)$ or using alternative notation.
This mark is given whether $(x+\delta x)^2 + 17(x+\delta x)$ stands alone or is embedded in an expression or a formula.

M1 For the <u>intent</u> to subtract $x^2 + 17x$ from the above.
So $(x+\delta x)^2 + 17(x+\delta x) - x^2 + 17x$ will gain the M1 even though there are missing brackets.
It can also be awarded to those who have expanded $(x+\delta x)^2 + 17(x+\delta x)$ and then crossed out the x^2 term and the $+17x$ term.
Those who reverse the subtraction will gain M0 <u>unless</u> there is evidence later on of dividing by $-\delta x$.

A1 For sight of $2x\delta x + (\delta x)^2 + 17\delta x$ (Accept δx^2 as meaning $(\delta x)^2$) with no other terms. Treat as a CAO.
$2x + \delta x + 17$ will imply the above if division by δx has already been done.

M1 A FT, if of equivalent difficulty, is possible for this M1 (but not the subsequent A1).
A <u>correct</u> division by δx has to be done
(so if a FT it has to be correct for their $2x\delta x + (\delta x)^2 + 17\delta x$)
<u>AND</u> we must see 'lim $\delta x \to 0$' OR '$\delta x \to 0$' OR 'δx tends to 0'.
It is M0 for '$\delta x = 0$' OR '$\delta x \approx 0$' OR ' δx is so small we can forget about it'.

All of the above marks can be gained even if there is no l.h.s. shown.

Final A1. Must be for a 'text book' quality presentation. E.g.
Has to be a correct l.h.s. for each line, 'δy' or '$\delta y/\delta x$'
AND at some point '$dy/dx = \lim_{\delta x \to 0} \delta y/\delta x$' or '$dy/dx = \lim_{\delta x \to 0} 2x + \delta x + 17$'