

2.3 Higher tier

Foundation tier content is in standard text.

Intermediate tier content which is in addition to foundation tier content is in underlined text.

Higher tier content which is in addition to intermediate tier content is in **bold** text.

**Candidates entered for GCSE Mathematics will be expected to be familiar with the knowledge, skills and understanding implicit in GCSE Mathematics – Numeracy.*

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<i>Understanding number and place value</i>	
Reading and writing whole numbers of any magnitude expressed in figures or words. Rounding whole numbers to the nearest 10, 100, 1000, etc. Understanding place value and decimal places. Rounding decimals to the nearest whole number or a given number of decimal places. <u>Rounding numbers to a given number of significant figures.</u>	
Using the equivalences between decimals, fractions, ratios and percentages. Converting numbers from one form into another. Ordering and comparing whole numbers, decimals, fractions and percentages. Understanding and using directed numbers, including ordering directed numbers.	
<i>Understanding number relationships and methods of calculation</i>	
Using the common properties of numbers, including odd, even, multiples, factors, primes. Expressing numbers as the product of their prime factors. <u>Least common multiple and highest common factor.</u> <u>Finding the LCM and HCF of numbers written as the product of their prime factors.</u> Using the terms square, square root, cube, <u>cube root and reciprocal.</u> The use of index notation for <u>zero, positive and negative</u> integral indices. <u>The use of index notation for positive unit fractional and other fractional indices.</u> Interpreting numbers written in standard form in the context of a calculator display. <u>Writing whole numbers in index form.</u> <u>Using the rules of indices.</u> <u>Expressing and using numbers in standard form with positive and negative powers of 10.</u>	

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<p>Using the facilities of a calculator, including the <u>constant function, memory and brackets</u>, to plan a calculation and evaluate expressions.</p> <p>Using addition, subtraction, multiplication, division, square, square root, <u>power, root, constant, memory, brackets and appropriate statistical functions</u>.</p> <p>Knowing how a calculator orders its operations. (Candidates will not be expected to list the key depressions that they have made.) Using calculators effectively and efficiently.</p> <p>Reading a calculator display correct to a specified number of decimal places or <u>significant figures</u>. <u>Using appropriate trigonometric functions on a calculator</u>.</p>	
<p>Understanding and using number operations and the relationships between them, including inverse operations and the hierarchy of operations.</p> <p>Addition, subtraction, multiplication and division of whole numbers, decimals, fractions and negative numbers.</p> <p>Finding a fraction or percentage of a quantity. Expressing one number as a fraction or percentage of another. Calculating fractional and percentage changes (increase and decrease), <u>including the use of multipliers</u>. <u>Repeated proportional changes; appreciation and depreciation</u>.</p> <p>Calculating using ratios in a variety of situations; proportional division. <u>Direct and inverse proportion</u>.</p> <p>The use of a non-calculator method to multiply and divide whole numbers up to and including the case of multiplication and division of a three-digit number by a two-digit number.</p>	
<p>Estimating and approximating solutions to numerical calculations. Using estimation in multiplication and division problems with whole numbers to obtain approximate answers, e.g. by first rounding the numbers involved to 1 significant figure. Candidates must show sufficient working in order to demonstrate how they have obtained their estimate.</p>	

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<p>Recognising that recurring decimals are exact fractions, and that some exact fractions are recurring decimals. Converting recurring decimals to fractional form.</p> <p>Distinguishing between rational and irrational numbers. Manipulating surds; using surds and π in exact calculations.</p> <p>Simplifying numerical expressions involving surds, excluding the rationalisation of the denominator of a fraction such as $\frac{1}{(2-\sqrt{3})}$.</p>	
<i>Solving numerical problems</i>	
<p>Interpretation and use of mathematical information presented in written or visual form when solving problems, e.g. TV programme schedules, bus/rail timetables, distance charts, holiday booking information.</p> <p>Money: The basic principles of personal and household finance, including fuel and other bills, hire purchase, discount, VAT, taxation, best buys, wages and salaries, loan repayments, mortgages, budgeting, exchange rates and commissions.</p> <p>Simple <u>and compound interest, including the use of efficient calculation methods.</u></p> <p>Profit and loss.</p> <p><u>Finding the original quantity given the result of a proportional change.</u></p> <p>Foreign currencies and exchange rates.</p> <p>Carrying out calculations relating to enterprise, saving and borrowing, investing, appreciation and depreciation and understanding annual rates, e.g. AER, APR.</p>	
<p>Giving solutions in the context of a problem, <u>selecting an appropriate degree of accuracy, interpreting the display on a calculator, and recognising limitations on the accuracy of data and measurements.</u></p> <p><u>Rounding an answer to a reasonable degree of accuracy in the light of the context.</u> Interpreting the display on a calculator. Knowing whether to round up or down as appropriate.</p>	

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<p><u>Recognising that measurement is approximate and that a measurement expressed to a given unit is in possible error of half a unit.</u> <u>The upper and lower bounds of numbers expressed to a given degree of accuracy.</u></p> <p><u>Calculating the upper and lower bounds in the addition and subtraction of numbers expressed to a given degree of accuracy.</u></p> <p>Calculating the upper and lower bounds in calculations involving multiplication and division of numbers expressed to given degrees of accuracy.</p>	
<p>Understanding and using Venn diagrams to solve problems.</p>	

Higher Tier - Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<i>Understanding and using functional relationships</i>	
<p>Recognition, description and continuation of patterns in number. Description, in words <u>and symbols</u>, of the rule for the next term of a sequence.</p>	<p><u>Finding the nth term of a sequence where the rule is linear or quadratic.</u> <u>Generating linear and non-linear sequences given the nth term rule.</u></p>
<p>Construction and interpretation of conversion graphs.</p> <p>Construction and interpretation of travel graphs. Construction and interpretation of graphs that describe real-life situations. Interpretation of graphical representation used in the media, recognising that some graphs may be misleading.</p> <p><u>Recognising and interpreting graphs that illustrate direct and inverse proportion.</u></p>	
<p>Using coordinates in 4 quadrants.</p> <p>Drawing, interpreting, <u>recognising and sketching</u> the graphs of $x = a$, $y = b$, $y = ax + b$.</p> <p><u>The gradients of parallel lines.</u></p>	<p><u>Identifying the equation of lines parallel or perpendicular to a given line, to satisfy given conditions.</u></p> <p><u>Knowledge and use of the form $y = mx + c$ to represent a straight line where m is the gradient of the line, and c is the value of the y-intercept.</u></p> <p><u>Drawing, interpretation, recognition and sketching the graphs of $y = ax^2 + b$, $y = \frac{a}{x}$, $y = ax^3$.</u></p> <p><u>Drawing and interpretation of graphs of $y = ax^2 + bx + c$, $y = ax^3 + b$.</u></p> <p><u>Drawing and interpretation of graphs of $y = ax + b + \frac{a}{x}$ with x not equal to 0, $y = ax^3 + bx^2 + cx + d$,</u></p> <p><u>$y = k^x$ for integer values of x and simple positive values of k.</u></p> <p><u>Drawing and interpreting graphs when y is given implicitly in terms of x.</u></p>

Higher Tier - Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
	<p>Understanding and using function notation. Interpreting and applying the transformation of functions in the context of their graphical representation, including $y = f(x + a)$, $y = f(kx)$, $y = kf(x)$ and $y = f(x) + a$, applied to $y = f(x)$.</p>
<p>Constructing and using tangents to curves to estimate rates of change for non-linear functions, and using appropriate compound measures to express results, including finding velocity in distance-time graphs and acceleration in velocity-time graphs.</p> <p>Interpreting the meaning of the area under a graph, including the area under velocity-time graphs and graphs in other practical and financial contexts.</p> <p>Using the trapezium rule to estimate the area under a curve.</p>	
<i>Understanding and using equations and formulae</i>	
<p>Substitution of positive and negative whole numbers, fractions and decimals into simple formulae expressed in words or in symbols.</p> <p>Understanding the basic conventions of algebra.</p> <p>Formation and simplification of expressions involving sums, differences, products and <u>powers</u>.</p> <p>Collection of like terms.</p> <p>Expansion of $a(bx + c)$, where a, b and c are integers.</p> <p>Formation, manipulation and solution of linear equations.</p> <p><u>Changing the subject of a formula when the subject appears in one term.</u></p>	<p><u>Extraction of common factors.</u></p> <p><u>Formation and manipulation of simple linear inequalities.</u></p> <p>Changing the subject of a formula when the subject appears in more than one term.</p> <p><u>Multiplication of two linear expressions; expansion of $(ax + by)(cx + dy)$ and $(ax + by)^2$, where a, b, c, d are integers.</u></p> <p><u>Factorisation of quadratic expressions of the form $x^2 + ax + b$ and $ax^2 + bx + c$, including the difference of two squares.</u></p> <p>Formation and manipulation of quadratic equations.</p> <p>Constructing and using equations that describe direct and inverse proportion.</p> <p>Simplifying algebraic fractions.</p>

Higher Tier - Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<p><u>The solution of linear equations with whole number coefficients in solving problems set in real-life contexts.</u></p>	<p><u>Solution of linear equations and linear inequalities with whole number and fractional coefficients.</u> The use of straight line graphs to locate regions given by linear inequalities.</p> <p><u>The formation and solution of two simultaneous linear equations with whole number coefficients by graphical and algebraic methods in solving problems set in real-life contexts.</u></p> <p><u>Solution by factorisation and graphical methods of quadratic equations of the form $x^2 + ax + b = 0$.</u> Solution by factorisation, graphical methods and formula, of quadratic equations of the form $ax^2 + bx + c = 0$, selecting the most appropriate method for the problem concerned. Solution of equations involving linear denominators leading to quadratic or linear equations.</p> <p><u>Solution of a range of cubic equations by trial and improvement methods, justifying the accuracy of the solution.</u></p>
	<p><u>Distinguishing in meaning between equations, formulae, identities and expressions.</u></p>

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<i>Understanding and using properties of shape</i>	
<p>The geometrical terms: point, line, plane, parallel, right angle, clockwise and anticlockwise turns, perpendicular, horizontal, vertical, acute, obtuse and reflex angles, face, edge and vertex.</p> <p>Vocabulary of triangles, quadrilaterals and circles: isosceles, equilateral, scalene, exterior/interior angle, diagonal, square, rectangle, parallelogram, rhombus, kite, trapezium, polygon, pentagon, hexagon, radius, diameter, tangent, circumference, chord, arc, sector, segment.</p> <p>Simple solid figures: cube, cuboid, cylinder, <u>prism</u>, <u>pyramid</u>, cone, sphere, <u>tetrahedron</u>.</p> <p>Interpretation and drawing of nets. Using and drawing 2-D representations of 3-D shapes, including the use of isometric paper.</p>	
<p>Accurate use of ruler, pair of compasses and protractor. (Lengths accurate to 2mm and angles accurate to 2°.)</p> <p>Bisecting a given line, bisecting a given angle.</p> <p><u>Constructing the perpendicular from a point to a line.</u></p> <p><u>Essential properties of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus; classify quadrilaterals by their geometric properties.</u></p> <p><u>Constructing 2-D shapes from given information and drawing plans and elevations of any 3-D solid.</u></p>	<p>Use of ruler and pair of compasses to do constructions.</p> <p>Construction of triangles, quadrilaterals and circles.</p> <p><u>Constructing angles of 60°, 30°, 90° and 45°.</u></p> <p><u>The identification of congruent shapes.</u></p> <p>Understanding and using SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments. Reasons may be required in the solution of problems involving congruent triangles.</p>
	<p>Simple description of symmetry in terms of reflection in a line/plane or rotation about a point.</p> <p>Order of rotational symmetry.</p>

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<p>Angles at a point. Angles at a point on a straight line. Opposite angles at a vertex.</p> <p>Parallel lines. Corresponding, alternate and interior angles.</p> <p>Angle properties of triangles. Using the fact that the angle sum of a triangle is 180°.</p>	<p>Using the fact that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices.</p> <p>Using angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of any quadrilateral is 360°.</p> <p>Regular and irregular polygons. Sum of the interior and sum of the exterior angles of a polygon.</p>
<p><u>Using Pythagoras' theorem in 2-D and 3-D, including reverse problems.</u></p>	
<p><u>Using trigonometric relationships in right-angled triangles to solve problems, including those involving bearings and angles of elevation and depression.</u> <u>Calculating a side or an angle of a right-angled triangle in 2-D and 3-D.</u></p>	
<p>Extending trigonometry to angles of any size.</p> <p>The application of trigonometric functions to the solution of problems in 2-D or 3-D, including appropriate use of the sine and cosine rules.</p> <p>Using the formula: area of a triangle = $\frac{1}{2}absinC$.</p>	<p>The graphs and behaviour of trigonometric functions.</p> <p>Sketching of trigonometric graphs.</p>
	<p><u>Using angle and tangent properties of circles.</u> <u>Understanding that the tangent at any point on a circle is perpendicular to the radius at that point.</u></p> <p><u>Using the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, that the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180°.</u></p> <p>Using the alternate segment theorem.</p> <p><u>Understanding and using the fact that tangents from an external point are equal in length.</u></p> <p>Understanding and constructing geometrical proofs using circle theorems.</p>

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<i>Understanding and using properties of position, movement and transformation</i>	
	<p>Finding the coordinates of points identified by geometrical information, for example, finding the coordinates of the mid-point of the line segment AB, given points A and B; finding the coordinates of the fourth vertex of a parallelogram, given the coordinates of the other three vertices.</p> <p>Location determined by distance from a given point and angle made with a given line.</p>
<p><u>Using the knowledge that, for two similar 2-D or 3-D shapes, one is an enlargement of the other.</u></p> <p><u>Using the knowledge that, in similar shapes, corresponding dimensions are in the same ratio.</u></p> <p>Using the relationships between the ratios of:</p> <ul style="list-style-type: none"> • lengths and areas of similar 2-D shapes, and • lengths, areas and volumes of similar 3-D shapes. 	<p>Transformations, including:</p> <ul style="list-style-type: none"> • Reflection • Rotation through 90°, 180°, 270°. Clockwise or anticlockwise rotations; centre of rotation • Enlargement with positive, fractional and negative scale factors • Translation; <u>description of translations using column vectors.</u> <p>Candidates will be expected to draw the image of a shape under transformation.</p> <p><u>Questions may involve two successive transformations.</u></p>
Solving problems in the context of tiling patterns and tessellation.	
<p>Using and interpreting maps. Interpretation and construction of scale drawings. Scales may be written in the form 1 cm represents 5 m, or 1:500.</p> <p>Use of bearings. (Three figure bearings will be used e.g. 065°, 237°.)</p>	
<p><u>Constructing the locus of a point which moves such that it satisfies certain conditions, for example,</u></p> <p><u>(i) a given distance from a fixed point or line,</u></p> <p><u>(ii) equidistant from two fixed points or lines.</u></p> <p><u>Solving problems involving intersecting loci in two dimensions.</u> <u>Questions on loci may involve inequalities.</u></p>	

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<i>Understanding and using measures</i>	
<p>Standard metric units of length, mass and capacity.</p> <p>The standard units of time; the 12- and 24- hour clock. (The notation for the 12- and 24- hour clock will be 1:30 p.m. and 13:30.)</p> <p>Knowledge and use of the relationship between metric units of length, mass, capacity, area and volume.</p> <p>Making sensible estimates of measurements in everyday situations, recognising the appropriateness of units in different contexts.</p> <p>Conversion between the following metric and Imperial units: km - miles; cm, m - inches, feet; kg - lb; litres - pints, gallons. Candidates will be expected to know the following approximate equivalences: 8km \approx 5 miles, 1kg \approx 2.2 lb, 1 litre \approx 1.75 pints</p>	
<p>Reading and interpreting scales, including decimal scales.</p>	
<p><u>Distinguishing between formulae for length, area and volume by considering dimensions.</u></p> <p>Using compound measures including speed, <u>density and population density.</u> Using compound measures such as m/s, km/h, mph, mpg, <u>kg/m³, g/cm³, population per km²</u></p>	
<p>Estimating of the area of an irregular shape drawn on a square grid.</p> <p>Calculating:</p> <ul style="list-style-type: none"> - perimeter and area of a square, rectangle, triangle, parallelogram, trapezium, circle, semicircle and composite shapes. - surface area, cross-sectional area and volume of cubes, cuboids, <u>prisms, cylinders and composite solids.</u> 	
<p>Lengths of circular arcs.</p> <p>Perimeters and areas of sectors and segments of circles.</p> <p>Surface areas and volumes of spheres, cones, pyramids and compound solids.</p>	

Higher tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using the statistical problem solving process: specifying the problem/planning; collecting, processing and representing data; interpreting and discussing results.	
<i>Specifying the problem and planning</i>	
<p>Specifying and testing hypotheses, taking account of the limitations of the data available. <u>Testing an hypothesis such as ‘Girls tend to do better than boys in biology tests’.</u></p> <p><u>Specifying the data needed and considering potential sampling methods.</u> <u>Sampling systematically.</u></p> <p>Working with stratified sampling techniques and defining a random sample.</p> <p>Designing and criticising questions for a questionnaire, including notions of fairness and bias.</p> <p><u>Considering the effect of sample size and other factors that affect the reliability of conclusions drawn.</u></p>	
<i>Processing, representing and interpreting data</i>	
<p>Sorting, classification and tabulation of qualitative (categorical) data, discrete or continuous quantitative data.</p> <p>Grouping of discrete or continuous data into class intervals of equal <u>or unequal</u> widths.</p> <p>Understanding and using tallying methods.</p>	
<p>Constructing and interpreting pictograms, bar charts and pie charts for qualitative data. Constructing and interpreting vertical line diagrams for discrete data.</p> <p>Constructing line graphs for the values of a variable at different points in time; understanding that intermediate values in a line graph may or may not have meaning.</p> <p>Constructing and interpreting grouped frequency diagrams <u>and frequency polygons.</u></p> <p>Temperature charts.</p> <p>Constructing and interpreting scatter diagrams for data on paired variables.</p> <p><u>Constructing and interpreting cumulative frequency tables and diagrams using the upper boundaries of the class intervals.</u></p>	

Higher tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
<p>Extending skills in handling data into constructing and interpreting histograms with unequal class widths. Frequency density. Interpreting shapes of histograms representing distributions (with reference to mean and dispersion).</p>	
<p><u>Selecting and using an appropriate measure of central tendency.</u> Mean, median and mode for a discrete (ungrouped) frequency distribution.</p> <p><u>Estimates for the median and mean of grouped frequency distributions.</u></p> <p>Comparison of two distributions using one measure of central tendency (i.e. the mean or the median) <u>and/or one measure of spread.</u></p> <p>Modal category for qualitative data. Modal class for grouped data.</p> <p><u>Estimating the median from a cumulative frequency diagram.</u></p> <p><u>Selecting and calculating or estimating appropriate measures of spread, including the range and interquartile range applied to discrete, grouped and continuous data.</u></p> <p><u>Producing and using box-and-whisker plots to compare distributions.</u></p> <p>Drawing 'by eye' a line of 'best fit' on a scatter diagram, understanding and interpreting what this represents. <u>[In questions where the mean point has been given, calculated or plotted, candidates will be expected to draw the line of 'best fit' through that point.]</u></p>	
Discussing results	
<p>Recognising that graphs may be misleading. Looking at data to find patterns and exceptions.</p> <p>Drawing inferences and conclusions from summary measures and data representations, relating results back to the original problem.</p> <p>Drawing of conclusions from scatter diagrams; using terms such as positive correlation, negative correlation, little or no correlation. Appreciating that correlation does not imply causality.</p>	

Higher tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Estimating and calculating the probabilities of events	
<p>Understanding and using the vocabulary of probability, including notions of uncertainty and risk.</p> <p>The terms 'fair', 'evens', 'certain', 'likely', 'unlikely' and 'impossible'.</p>	<p>Understanding and using the probability scale from 0 to 1.</p> <p>Use of: the probability of an event not occurring is one minus the probability that it occurs. (Probabilities must be expressed as fractions, decimals or percentages.)</p>
	<p>Estimating the probability of an event as the proportion of times it has occurred.</p> <p>Relative frequency.</p> <p>An understanding of the long-term stability of relative frequency is expected. <u>Graphical representation of relative frequency against the number of trials.</u></p> <p>Calculating theoretical probabilities based on equally likely outcomes.</p> <p>Estimating probabilities based on experimental evidence.</p> <p>Comparing an estimated probability from experimental results with a theoretical probability.</p>
	<p>Identifying all the outcomes of a combination of two experiments, <i>e.g. throwing two dice</i>; use tabulation, <u>tree diagrams</u>, Venn diagrams, or other diagrammatic representations of compound events.</p>
	<p>Knowledge that the total probability of all the possible outcomes of an experiment is 1.</p> <p><u>Recognising the conditions when the addition of probabilities for mutually exclusive events and the multiplication of probabilities for two independent events apply, and making the appropriate calculations.</u></p> <p><u>If A and B are mutually exclusive, then the probability of A or B occurring is $P(A) + P(B)$.</u></p> <p><u>If A and B are independent events, the probability of A and B occurring is $P(A) \times P(B)$.</u></p>
	<p>Understanding when and how to estimate conditional probabilities.</p> <p>The multiplication law for dependent events.</p> <p>Sampling without replacement.</p>