

1)

The curve  $C$  has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i)  $\frac{dy}{dx}$

(ii)  $\frac{d^2y}{dx^2}$

(3)

(b) Verify that  $C$  has a stationary point when  $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

2)

A curve has equation  $y = f(x)$  and is such that  $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$ .

(i) By using the substitution  $u = x^{\frac{1}{2}}$ , or otherwise, find the values of  $x$  for which the curve  $y = f(x)$  has stationary points. [4]

(ii) Find  $f''(x)$  and hence, or otherwise, determine the nature of each stationary point. [3]

(iii) It is given that the curve  $y = f(x)$  passes through the point  $(4, -7)$ . Find  $f(x)$ . [4]

3)

The curve  $C$  has equation  $y = 6 - 3x - \frac{4}{x^3}$ ,  $x \neq 0$

(a) Use calculus to show that the curve has a turning point  $P$  when  $x = \sqrt{2}$  [4]

(b) Find the  $x$ -coordinate of the other turning point  $Q$  on the curve. [1]

(c) Find  $\frac{d^2y}{dx^2}$ . [1]

(d) Hence or otherwise, state with justification, the nature of each of these turning points  $P$  and  $Q$ . [3]

4)

- (i) Find the coordinates of the stationary point on the curve  $y = x^4 + 32x$ . [5]
- (ii) Determine whether this stationary point is a maximum or a minimum. [2]

5)

A curve has equation  $y = (x + 2)(x^2 - 3x + 5)$ .

- (i) Find the coordinates of the minimum point, justifying that it is a minimum. [8]
- (ii) Calculate the discriminant of  $x^2 - 3x + 5$ . [2]
- (iii) Explain why  $(x + 2)(x^2 - 3x + 5)$  is always positive for  $x > -2$ . [2]

6)

The curve  $C$  has equation  $y = 12\sqrt{(x) - x^{\frac{3}{2}} - 10}$ ,  $x > 0$

- (a) Use calculus to find the coordinates of the turning point on  $C$ . (7)
- (b) Find  $\frac{d^2y}{dx^2}$ . (2)
- (c) State the nature of the turning point. (1)